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Process Mean and Production Policies Determination under Constant Supply of Perishable Raw Material

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ABSTRACT

Selection of the mean (target value) for a container-filling process is an important decision to a producer especially when material cost is a significant portion of production cost. Because the process mean determines the process conforming rate, it affects other production decisions, including, in particular, production setup and raw material procurement policies. It is evident that these decisions should be made jointly in order to control the production, inventory and raw material costs. In this paper, we consider product of interest is assumed to have a lower specification limit, and the items that do not conform to the specification limit are scrapped with no salvage value. The production cost of an item is a linear function of the amount of the raw material used in producing the item, and the supply rate of the raw material is finite and constant. Furthermore, it is assumed that the raw material is perishable and the perishability is incorporate into an existing model that was developed for simultaneously determining the process mean, production setup and raw material procurement policies for a container-filling process. A two-echelon model is formulated for a single-product production process, and an efficient algorithm is developed for finding the optimal solution.

Keywords: inventory theory, purchasing, perishability

I. INTRODUCTION

Selection of the process mean (target value) for a container-filling process is a classical problem in quality control. A typical situation considered in this problem is as follows. The product of interest has a performance variable with a lower specification limit, and the raw material requirement for producing the product is an increasing function of the performance variable. An item is classified as conforming if its value of performance variable is larger than or equal to the lower limit. Otherwise, the item is a nonconforming item. The economical components considered in determining the process mean include: (1) the payoff (revenue) associated with the conforming items, (2) the payoff (revenue

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or cost) associated with the disposition of nonconforming items, and (3) the production (raw material) cost. The optimal process mean is determined by maximizing the expected total profit or minimizing the expected total cost.

The existing studies can be categorized into three areas. The focus of the first area is on the payoff aspects of the conforming and nonconforming items. For example, Hunter and Kartha (1977) discussed the situation in which nonconforming (underfilled) items can be sold at a reduced price and a penalty is incurred by the conforming items with excess quality (the difference between the performance variable and the lower limit).

The second area focuses on reprocessing nonconforming items as well as overfilled items. For example, Golhar (1987) assumed that only the regular market (fixed selling price) is available for the conforming items and that the underfilled items are reprocessed. Golhar and Pollock (1988) extended Golhar's (1987) model to include an artificial upper limit so that nonconforming items are re-processed, as well as the items larger than the upper limit.

The models in the first two areas assume implicitly or explicitly that a screening (100% inspection) procedure is used to measure the performance variable in order to determine the selling prices and/or the corrective actions. In the third area, different inspection methods are considered. For example, Tang and Lo (1993) discussed a situation in which a surrogate variable is used as the screening variable. In addition, variables and attributes acceptance sampling plans have also been considered (Boucher and Jafari, 1991; Carlsson, 1980). Tang and Tang (1994) have given a detailed review of these three areas.

Because the process mean determines the process conforming rate, the former affects the production yield rate. As a result, other production decisions, including, in particular, production setup and raw material procurement policies, are also dependent on the process mean setting. Roan, Gong and Tang (1994) incorporate the issues associated with production setup and raw material procurement into the process mean problem. It is assumed that the product of interest requires one major raw material, which is purchased from outside vendors with a fixed unit cost. The production cost of an item is a linear function of the amount of the raw material used in producing the item. A two-echelon model is formulated for jointly determining the process mean, production lot size, and raw material procurement policy in a single-product production process. Roan, Gong and Tang (1997) consider a situation wherein a constant supply rate of raw material is offered by a reliable source. Instead of deciding the optimal raw material order quantity, setting the optimal supply rate becomes the key concern between the manufacturer and the supplier.

All the papers mentioned above ignore a practical situation, that is, the deterioration of the raw material or items produced. Deterioration is applicable to many inventory items in practice, such as volatile liquids, agriculture products, radioactive substances, drugs, blood, fashion goods, electronic components, and high-tech products. Existing inventory models for deterioration items are numerous, and can be divided into two categories: fixed lifetime and random lifetime. The first category includes those cases where the life time is known a priori to be a specified number of periods or a length on time independent of all other parameters of the system. While the second category include exponential decay as a special case and will also include those cases where the product lifetime is a random variable with specified probability distribution.

Ghare and Schrader (1963) developed an EOQ model wherein they have assumed exponential decay of items (i.e. constant rate of deterioration over time). Emmons (1968) considered the decay of radioactive nuclide generators. This differs from the usual consideration in that decay is also the total usage. Covert and Philip (1973), Philip (1974) and Misra (1975) generalized Ghare and Schrader's model in the case of deterioration governed by a Weibull distribution. Further extensions of Ghare and Schrader's model were considered by Shah (1977) and Tadikamalla (1978). Tadikamalla assumed that the lifetime of items is governed by a gamma rather than Weibull distribution while Shah considered the case of an arbitrary distribution.

In this paper, we extend the model proposed by Roan et al. (1997) to consider a very common situation in practice, that is, raw material is deteriorated over time following an exponential distribution.

The organization of the paper is as follows. The assumptions and model formulation are given in the next section. Then, in section 3, analytical properties of the optimal solution are derived and a solution algorithm is proposed. An example is also given to illustrate the solution procedures in section 3. A sensitivity analysis on the effects of model parameters on the optimal solution and the effect of deterioration comparing to the original model is presented in section 4. The last section is a brief summary of the results given in this paper and possible future extensions.

II. MODEL ASSUMPTION AND FORMULATION

2.1 Notations and Assumptions

2.1.1 Notations

In addition to the notations described in the content, the notations are summarized as follow.

D is demand rate per unit time, which is assumed constant.

r is production rate per unit time, which is assumed constant.

X is the performance variable of interest.

L is the lower specification limit of X.

μ is an adjustable mean of the production process, which is a decision variable.

σ^2 is a constant variance of the production process, which is a constant.

$f(x)$ is the probability density function of X.

$\Phi(\cdot)$ is the standard normal distribution function.

p is the conforming rate of the production process.

c is the unit cost of the raw material.

b is the fixed production cost.

α is a constant larger than or equal to 1.

$g(\cdot)$ is the direct cost of producing an item

h is the holding cost of each unit of raw material per unit time.

h_1 is the holding cost of a monetary unit of raw material per unit time.

H is the cost of holding a conforming item for a unit time.

q is the production run size, which is a decision variable.

S is the production setup cost.

β is the raw material supply rate per unit time, which is a decision variable.

θ is the raw material deteriorating rate.

I_t is the amount of raw material surviving to time t exclusive of demand.

$d(\cdot)$ is the consumption rate of the raw material used in production.

2.1.2 Assumptions

Consider a product with a constant demand rate of *D* items per unit time. A production process with a production rate of r items per unit time is used to satisfy the demand. Let *X* denote the performance variable of interest, which is a measure of the raw material used in the production, such as weight and volume. Let *L* be the lower specification limit of *X*, so that an item is conforming if its *X* value is larger than or equal to *L*. Assume that the production process is stable and *X* follows a normal distribution with an adjustable mean μ and a constant variance σ^2. For given μ, the conforming rate of the production process is

$$p = \int_{L}^{\infty} f(x)dx = 1 - \Phi\left(\frac{L-\mu}{\sigma}\right), \tag{1}$$

where $f(x)$ is the probability density function of *X* and $\Phi(\cdot)$ is the standard normal distribution function.

Assume that nonconforming items are scrapped with no salvage value. Consequently, for given μ, the yield of the production process is $r \times p$ items per unit time. It is assumed that all the demand will be satisfied in such a way that the expected total number of conforming items produced is equal to the total demand and no backlog is allowed. Note that use of the expected conforming items is reasonable, especially in high-speed production, because the production output can be treated as approximately constant. Otherwise, it may require considering safety stock or using other inventory models. Note that $r \times p$ has to be greater than or equal to *D* to ensure that the production capacity is large enough to meet the demand.

Suppose the unit cost of the raw material is *c*; thus, *cX* is the material cost required for producing an item of the finished product. We further assume that the direct cost of producing an item is a linear function of the item's material cost:

$$g(X) = b + \alpha cX,$$

where *b* is the fixed production cost, and α is a constant larger than or equal to 1.

This cost function indicates that the production cost consists of a fixed cost and a variable cost that is proportional to the raw material used in production. This cost function allows us to model the production cost for different container-filling process. The fixed cost, *b*, may include the cost of a container, the average production cost excluding the raw material cost, and the overhead cost. Furthermore, if the raw material is processed before the filling process, the value of the material will increase. In this situation, α-1 is the relative value (cost) added on the raw material during the production process. It can be verified that, for given μ, the expected cost of yielding a conforming item is $(b+\alpha c\mu)/p$. Let *h* be the cost of holding each unit of the raw material for a unit time. In other words, the cost of holding a monetary unit of raw material is $h_1 = h/c$ per unit time. Assume that the costs of holding a monetary unit of raw material and finished product are the same. Then, the cost of holding a conforming item for a unit time is

$$\mathbf{H = \frac{h}{p}\left(\alpha\mu + \frac{b}{c}\right).}$$

Let *q* be the production run size, which is the number of items (including both conforming and nonconforming items) produced in a production run. The inventory level as a function of time is described in Figure 1(a). Assume that a production run begins at time 0. The finished product inventory increases at a rate of rp-D items per unit time until q items are produced. At time q/r, the production run is complete, and, then, the inventory decreases at a rate of *D* items per unit time until time qp/D when the inventory level reaches 0 and the second production run starts. Let *S* be the production setup cost. Since the total number of setups required per unit time is D/qp, the total setup cost is SD/qp. It can be verified that the average inventory level for the finished product is $(q/2r)(rp-D)$. As a result, the total holding cost for finished products is $H(q/2r)(rp-D)$ per unit time. Furthermore, because the expected cost of yielding a conforming item is $(b+\alpha c\mu)/p$, the per-unit-time direct production cost is $D(b+\alpha c\mu)/p$.

2.2 Model Formulation

We define the cost associated with the finished product for given raw material unit cost *c* as the sum of the production cost, the process setup cost, and the inventory holding cost:

$$\mathbf{FPC(\mu,q) = \frac{D(b+\ \alpha c\mu)}{p} + \frac{DS}{pq} + H\frac{q}{2r}(rp - D).} \quad \textbf{(2)}$$

The expected amount of the raw material required to produce one conforming item is μ/p. It is assumed that the raw material arrives at a constant rate β per unit time, regardless of whether the production process is in operation or is idle. Assume all the raw material received by the producer is either used in production or deteriorated over time. Suppose the lifetime of raw material is a random variable with an exponential distribution having parameter θ. Let I_T be the amount of raw material surviving to time t exclusive of demand (used in production). The expected amount of raw material surviving until time $T + dt$ is $I_T e^{-\theta dt}$. In other words, within time dt, the consumption of the raw material is the sum of the demand usage $d(t)dt$, which used in production, and $I_T\,(1\text{-}e^{-\theta dt})$, which deteriorated over time. That is

$$-dI = I_T\,(1-e^{-\theta dt}) + d\,(t)\,dt \quad \textbf{(3)}$$

This can be transformed into a differential equation

$$\frac{dI}{dt} + I_T\,\theta = -d(t). \quad \textbf{(4)}$$

Solving the differential equation, we can get

$$I_T = I_0\,e^{-\theta T} - e^{-\theta T}\int_0^T d(t)e^{\theta t}\,dt \quad \textbf{(5)}$$

or

$$I_0 = I_T\,e^{\theta T} + \int_0^T d(t)e^{\theta t}\,dt \quad \textbf{(6)}$$

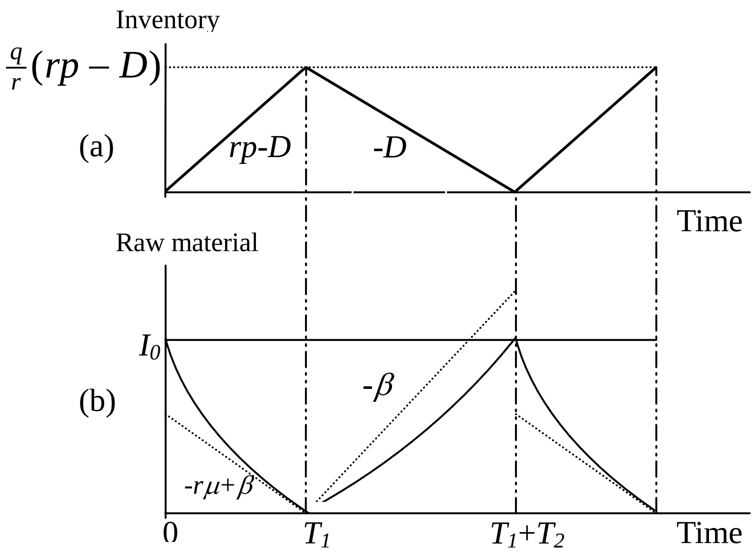

Figure 1. (a) Inventory Level of Finished Product;
(b) Inventory Level of Perishable Raw Material

In Figure 1(b), the inventory level of the perishable raw material as a function of time is shown. The dotted line, on the other hand, shows the inventory level of non-perishable raw material.

We assume that the initial raw material inventory level at time 0 is I0, which is sufficient to meet the requirement in the first production run. When production starts, the raw material inventory decreases at a rate of (-rμ+β) plus the deterioration. At time T_1 (i.e. $\frac{q}{r}$) the raw material inventory is depleted and the first production run is finished. Then, the process is idle, and the raw material inventory level increases at a rate of β and deteriorates at a rate of θ until the second production run starts at time (T_1+T_2)(i.e. $\frac{pq}{D}$). In other words, the inventory levels at time 0 and time (T_1+T_2) should be the same. That is

$$\boldsymbol{I_0} = \frac{(r\mu-\beta)}{\theta}(e^{\theta T_1}-1) = I_{T_1+T_2} = \frac{\beta}{\theta}(1-e^{-\theta T_2}) \qquad \textbf{(7)}$$

Substituting T_1 and T_2 and ignoring terms with second and higher order powers of θ, we can find

$$\beta = \frac{D\mu}{p}$$

or

$$\mu = \frac{p\beta}{D}. \tag{8}$$

In order to determine the total holding cost per unit time for the raw material, we have to find the average raw material inventory level first. From Figure 1(b), the average raw material inventory level is

$$\{(r\mu - \beta)(\frac{q}{2r^2} + \frac{\theta q^2}{6r^3}) + \beta[\frac{q}{2}(\frac{p}{D} - \frac{1}{r})^2 + \frac{\theta q^2}{6}(\frac{p}{D} - \frac{1}{r})^3]\} \times \frac{D}{p}.$$

The total holding cost per unit time for the raw material is

$$MC(\beta,q) = \{\{(r\mu - \beta)(\frac{q}{2r^2} + \frac{\theta q^2}{6r^3}) + \beta[\frac{q}{2}(\frac{p}{D} - \frac{1}{r})^2 + \frac{\theta q^2}{6}(\frac{p}{D} - \frac{1}{r})^3]\} \times \frac{D}{p}\} \times h_1 c \tag{9}$$

Substituting eq. (8) into eq. (2), eq. (2) can then be expressed as a function of β and q, and the total cost is the sum of $FPC(\beta,q)$ and $MC(\beta,q)$:

$$\begin{aligned} TC(\beta,q) &= FPC(\beta,q) + MC(\beta,q) \\ &= \alpha\beta c + \frac{Db}{p} + \frac{DS}{pq} + h_1\left(\frac{\alpha\beta c}{D} + \frac{b}{p}\right)\frac{q}{2r}\left(rp - D\right) + \\ &\quad \{\{\frac{1}{D}(rp - D)(\frac{q}{2r^2} + \frac{\theta q^2}{6r^3}) + [\frac{q}{2}(\frac{p}{D} - \frac{1}{r})^2 + \frac{\theta q^2}{6}(\frac{p}{D} - \frac{1}{r})^3]\} \times \frac{D}{p}\} \times h_1 c\beta \end{aligned} \tag{10}$$

III. OPTIMAL SOLUTION

In this section, we first derive important analytical property for the optimal solution for the problem defined in the last section. Based on the result, efficient solution algorithm is proposed to find the optimal process mean, production run size, and material supply rate.

3.1 Analytical Properties

For given β, the optimal production run size is found by solving $\partial TC(\beta,q)/\partial q=0$.

Result 1:

That is, q satisfies the following equation.

$$\frac{H}{2r}(rp-D)q^2-\frac{DS}{p}+\{\frac{1}{D}(rp-D)(\frac{q^2}{2r^2}+\frac{\theta q^3}{3r^3})+[\frac{1}{2}(\frac{p}{D}-\frac{1}{r})^2q^2+\frac{\theta q^3}{3}(\frac{p}{D}-\frac{1}{r})^3]\}\frac{D}{p}h_1c(\beta)=0 \quad \textbf{(11)}$$

The above equation is a cube function of q. It's not easy to solve manually. However, it can be solved by coding and using routines for solving nonlinear equations in IMSL MATH/LIBRARY.

3.2 Solution Algorithm

Because the raw material supply rate is a function of the process mean and Result 1 gives the optimal production run size corresponding to a given process mean, the total cost becomes a function of the process mean. Therefore, a one-dimensional direct search, such as golden-section search method or any other bi-section search method, can be used to search for the optimal process mean. In this paper, the range for the search is [L, L+4σ] and the search is terminated when the width of the interval of uncertainty is less than or equal to 0.0001. In most applications, 4σ above L is large enough to include the possible optimal solution. The reason L is used as the lower bound is that when μ equals L, the process conforming rate is 50%, which is very low in most realistic applications. Note that the golden-section search is based on the splitting of a line into two segments, which were actually known in ancient times as the "golden section." The ratio of the whole to the larger segment is equal to the ratio of the larger segment to the smaller segment. The golden-section search method provides the optimal solution if the objective function is unimodal, which was found to be true in all the examples that we tested. In general, multiple starting points can always be used in the search procedure to ensure that the global minimum is found.

The analytical result presented above provides the basis for developing efficient solution procedure to find the optimal solution for the problem. The solution procedure is given as follows.

Step 1. For given μ, β equals $D\mu/p$.

Step 2. When β is known, q satisfies eq. (10) in Result 1.

Step 3. For all the possible pairs of (β,q), computes total cost $TC(\beta,q)$.

Step 4. If the total cost found in Step 3 less than the lowest total cost found previously, replaces it.

Step 5. If termination condition is meet, the μ, β and q corresponding to lowest total cost are optimal process mean, raw material supply rate and production lot size, respectively. Otherwise, go to Step 1.

A FORTRAN program has been written for implementing the solution procedures on an Pentium III computer. The running time for solving each of the problems considered in this paper is only a few seconds.

3.3 Example

Here, an example, same as the one used in Roan *et. al.* (1997), is used to illustrate the solution procedures given in the last section. The example will also be used in the sensitivity analysis in Section 5.

Consider a product that requires at least 8 mgs of main content in each item. Any item that is less than 8 mgs is considered nonconforming and is scrapped without salvage value. Because of the variation in the production process, the content of an item produced by the process follows a normal distribution, with an adjustable process mean and a constant standard deviation of 1.2 mgs. Assume that the product demand rate and the production rate are 5,000 items and 7,500 items per unit time, respectively. The setup cost per production run is $150, the fixed production cost is $0.16 per item, and α is 4. The raw material is constantly supplied by a vendor at a rate determined by the producer. Furthermore, the cost for holding $1 of inventory (finished product or raw material) is $0.03 per unit time. The material cost is $0.1 per mg. The deterioration rate θ is 0.1.

The golden-section search method is used to find the solution here. The result of the algorithm is given as follows.

Step 1. When μ = 9.916, p = 0.9449 and the raw material supply rate β = 52,475.09.

Step 2. There is only one real solution found. That means q = 6,336.

Step 3. TC(52475.09,6336) = 22,105.87.

Step 4. 22105.87 less than the lowest total cost found previously, replaces it.

Step 5. 22105.87 is the lowest total cost until the search procedure terminated. The corresponding μ, β and q are optimal.

Therefore, the optimal process mean is found to be 9.916 mgs. The corresponding raw material supply rate and the production run size are 52,475.09mgs and 6,336 items, respectively. The total cost is $22,105.87. The process conforming rate is 94.49%.

IV. SENSITIVITY ANALYSIS

In this section, a sensitivity analysis is performed to study the effects of the model parameters on the optimal solutions. The model parameters included in the study are the deterioration rate θ, the demand rate D, the production rate r, the process standard deviation σ, the value-added factor α, and the production setup cost S. The sensitivity analyses are based on the example given in the last section.

4.1 Effect of Deterioration Rate

Table 1 Effect of Deterioration Rate

θ	μ^*	q^*	β^*	*FPC*	*MC*	*TC*
.00	9.915	5927	52475	22077.47	27.03	22104.50
.05	9.9153	6344	52475	22077.47	28.10	22105.57
.10	9.916	6336	52475	22077.47	28.40	22105.87
.20	9.916	6322	52475	22077.49	28.98	22106.47
.30	9.916	6308	52475	22077.52	29.55	22107.07
.40	9.916	6294	52475	22077.54	30.12	22107.66
.50	9.916	6281	52475	22077.57	30.69	22108.26
.60	9.916	6267	52475	22077.59	31.25	22108.84
.70	9.916	6254	52475	22077.62	31.81	22109.43
.80	9.916	6241	52475	22077.65	32.36	22110.01

θ: Raw material deterioration rate
μ^*: Optimal process mean
q^*: Optimal production lot size
β^*: Optimal raw material supply rate
FPC : Finished product cost
MC : Material cost
TC : Total cost

The deterioration rate studied here ranges from 0.0 through 0.8, with an increment of 0.1. Table 1 contains the optimal solutions for the selected deterioration rates, including the model parameters, μ, q, and β, and the total costs associated with the optimal solution.

Contrary to what one would expect, the optimal process mean remains almost the same when deterioration rate changes. As a result, the raw material supply rate keeps stable. However, the optimal production lot size increases when deterioration exists and decreases as deterioration rate increases. Because of the deterioration, it takes longer time to accumulate the raw material needed for a production run. As a result, the production lot size increases in order to reach a higher inventory level when production stops, therefore, to satisfy the demand while raw material is accumulated.

As one would intuitively expect, the total cost increases when deterioration rate increases. However, the increment is small. This is because the increment incurred mainly due to the deterioration of the raw material. In our model, compare to the finished product related cost, the raw material related cost is considered to be small. Therefore, even the cost increment is small, the increment percentage is considered to be large.

4.2 Effect of Demand Rate

To study the effects of demand rate, we use the results for selected values of D ranging from 1,500 to 7,000 per unit time. As shown in Table 2, when D increases, the optimal production run size for cases 1 (without deterioration) and 2 (with deterioration) increases. This result makes intuitive sense. When the demand rate is very low relative to the production rate (process capacity), no clear patterns were observed on the optimal process mean. When the demand rate is moderate or very high, both the optimal process mean and the material supply rate increase steadily as the demand rate increases. When the demand rate is very close to the production rate, because of a low accumulation rate of finished products, the production run size becomes very large and sensitive to the change of D.

The optimal process means under different demand rate are almost the same for both cases, while the production lot sizes in case 2 are larger than those in case

1. This is because the inventory carried need to be high enough to satisfy the demand during the time needed to accumulate the raw material for next production run.

Table 2 Effect of Demand Rate

D	CASE 1			CASE 2		
	μ*	*q**	β*	μ*	*q**	β*
1500	9.921	1982	15743	9.923	2111	15743
2000	9.923	2399	20990	9.921	2558	20990
2500	9.919	2825	26238	9.922	3015	26238
3000	9.920	3278	31485	9.921	3501	31486
3500	9.918	3781	36733	9.919	4040	36733
4000	9.918	4356	41980	9.918	4657	41980
4500	9.916	5049	47228	9.917	5398	47228
5000	9.915	5927	52475	9.916	6336	52475
5500	9.912	7136	57722	9.914	7623	57722
6000	9.909	9019	62969	9.910	9633	62969
6500	9.903	12854	68216	9.333	682645	69996
7000	9.801	1609350	73510	9.802	356467	73509

D : Product demand rate
CASE 1 : Raw material without deterioration
CASE 2 : Raw material with deterioration

4.3 Effect of Production Rate

Table 3 contains the results under selected values of r ranging from 6,000 to 10,000. The results of the both cases are very similar. It is clear that a large production rate results in a smaller *q** and a larger μ*. A larger μ* implies a higher production yield rate and a higher per-item holding cost. In order to avoid carrying a high level of finished product inventory, therefore, the manufacturer should lower the production run size, so as to reduce the cost of holding finished product inventory. Similar to the situation under the change of demand rate, the optimal process means under different production rate are almost the same for both cases, while the production lot sizes in case 2 are larger than those in case 1.

Table 3 Effect of Production Rate

r	CASE 1			CASE 2		
	μ*	*q**	β*	μ*	*q**	β*
6000	9.905	9410	52474	9.904	9453	52474
6500	9.910	7473	52474	9.910	7504	52474
7000	9.914	6514	52475	9.913	6541	52475
7500	9.915	5927	52475	9.916	5951	52475
8000	9.917	5526	52475	9.918	5548	52475
8500	9.917	5233	52475	9.918	5054	52475
9000	9.917	5009	52475	9.918	5355	52475
9500	9.919	4829	52476	9.919	5164	52476
10000	9.919	4684	52476	9.920	5007	52476

r : Production rate

4.4 Effect of Process Standard Deviation

Table 4 Effect of Process Variation

σ	CASE 1				CASE 2			
	μ*	*p*	*q**	β*	μ*	*p*	*q**	β*
.10	8.266	.9961	5946	41492	8.266	.9961	6353	41492
.50	9.005	.9779	5870	46048	9.006	.9779	6274	46048
1.00	9.689	.9543	5893	50760	9.688	.9543	6301	50760
2.00	10.640	.9066	6167	58682	10.640	.9066	6595	58682
3.00	11.203	.8573	6750	65349	11.205	.8573	7214	65349
4.00	11.410	.8031	7906	71043	11.411	.8031	8450	71043
5.00	11.140	.7349	11302	75782	11.138	.7349	12080	75783
6.00	10.584	.6667	1435473	79383	10.584	.6667	1153772	79383

σ : Process standard deviation

It is well known that the performance of a process can be improved by reducing its inherent variation. For a given process mean, a small process standard deviation implies a higher process yield rate. On the other hand, to maintain the same yield rate, the process mean can be set lower when σ is smaller. In this situation, the material requirement is reduced and thus the material ordering policy may be also affected. To study the effect of the process standard deviation on the optimal solution, the optimal solutions for some selected values of σ ranging from .1 to 6.0 are reported in table 4.

As one would expected, the total cost decreases as σ decreases. When σ increases, the process conforming rate decreases even though the process mean increases until σ = 4.0. The decrease in the conforming rate is mainly because of process variation and excess capacity. The conforming rate becomes very stable, however, when the process yield rate is close to the demand rate. The production run size is relatively stable when σ is small, and becomes very sensitive to σ when the production yield rate is close to the demand rate. The material ordering policy is relatively less sensitive to σ. It shows, however, that material order quantity increases when σ increases.

4.5 Effect of Value-Added Factor

As shown in Table 5, as α increases, the process mean first decreases and then remains stable while production lot size keeps decreasing. The main reason for this is that a larger α implies a larger cost of producing an item and so is the holding cost. In order to reduce these costs, a lower process mean is used.

Table 5 Effect of Value-Added Factor

α	CASE 1			CASE 2		
	μ^*	q^*	β^*	μ^*	q^*	β^*
1.0	9.981	9037	52498	9.981	10744	52498
2.0	9.937	7538	52479	9.939	8438	52480
3.0	9.923	6591	52476	9.923	7168	52476
4.0	9.915	5927	52475	9.916	6336	52475
5.0	9.911	5430	52474	9.910	5741	52474
6.0	9.908	5040	52474	9.909	5284	52474
7.0	9.906	4723	52474	9.906	4923	52474
8.0	9.905	4459	52474	9.904	4627	52474
9.0	9.903	4236	52474	9.904	4377	52474

α : Value-added factor

The above scenario happened in both cases 1 and 2. The difference between the production lot sizes decreases as α increases. The reason is the increase of holding cost is more than the saving of setup cost. As a result, the production cycle is decreased.

4.6 Effect of Production Setup Cost

When the production setup cost increases, the production run size is expected to increase in order to reduce the number of production setups. This result is observed in Table 6. It is interesting, however, to observe that the optimal process mean decreases as the production setup cost increases. Note that a large production run size results in holding a larger inventory, and that a smaller process mean, on the other hand, reduces per-item inventory cost and process yield rate. Since the decreases in the process means are small, the process yield rate remains quite stable. Furthermore, the material supply rate is also very stable when the production setup cost increases. Consequently, it can be concluded that only the production run size is significantly affected by the production setup cost.

Table 6 Effect of Production Setup Cost

S	CASE 1			CASE 2		
	μ*	*q**	β*	μ*	*q**	β*
50	9.919	3419	52475	9.921	3638	52476
100	9.918	4837	52475	9.918	5173	52475
150	9.915	5927	52475	9.916	6336	52475
200	9.915	6845	52475	9.914	7317	52475
250	9.911	7660	52474	9.912	8181	52475
300	9.910	8392	52474	9.911	8962	52475
350	9.910	9064	52474	9.909	9682	52474
400	9.908	9694	52474	9.909	10348	52474
450	9.907	10284	52474	9.908	10976	52474
500	9.906	10843	52474	9.906	11572	52474
550	9.905	11375	52474	9.906	12135	52474

S : Production setup cost

V. CONCLUSION

In this paper, it is assumed that the raw material is supplied at a constant rate from outside vendors. Two cases in terms of deterioration in the raw material are introduced into a two-echelon model, which incorporates the issues associated with production run size and raw material procurement policy into the classical process mean problem for a single-product production process. The performance variable of the product has a lower specification limit, and the items that do not conform to the specification limit are scrapped with no salvage value. The production cost of an item is a linear function of the amount of raw material used in producing the item. Two situations (without and with deterioration) are considered in the raw material. An efficient solution procedure has been developed for a joint determination of process mean, production run size and material procurement policy for minimizing the total cost incurred by production, inventory holding and raw material procurement and deterioration. A sensitivity analysis reveals the effect of the model parameters on the optimal solutions under two cases.

The effects of perishability cannot be disregarded in many inventory systems. This study found that the optimal process mean and optimal supply rate of raw material is less sensitive to the perishability, while optimal production run size increases when raw material is perishable.

The model structure presented in this paper provides a useful framework for future research on several interesting issues related to this classical problem in quality control. In particular, the following several extensions are possible. First, the deterioration of the production process can be incorporated into the model. Compare the effects caused by deterioration of finished products and/or raw material would be another possible extension. The third extension is to consider a multiple-level filling process in which several raw materials are added in different stages. However, this issue could become very complicated when the product conformance is jointly determined by the amount of several raw materials. These issues have been included in the authors' future research plans.

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APPENDIX

Proof of eq. (7):

At time T_1 (i.e. $\frac{q}{r}$) the raw material inventory is depleted and the first production run is finished, that is $I_{T_1}=0$ and $d(t)=(r\mu-\beta)$ between time 0 and time T_1. From eq. (6),

$$I_0 = \int_0^{T_1} d(t)e^{\theta t}dt$$

$$= d(t)\int_0^{T_1} e^{\theta t}dt$$

$$= d(t)\left(\frac{e^{\theta t}}{\theta}\right)\Big|_0^{T_1}$$

$$= \frac{(r\mu-\beta)}{\theta}(e^{\theta T_1}-1)$$

Then, the process is idle, and the raw material inventory level increases at a rate of β and deteriorates at a rate of θ until the second production run starts at time (T_1+T_2)(i.e. $\frac{pq}{D}$), that is $I_{T_1}=0$ and $d(t)=-\beta$ from time T_1 and time (T_1+T_2). From eq. (5),

$$I_{T_1+T_2} = -e^{-\theta T_2}\int_0^{T_2} d(t)e^{\theta t}dt$$

$$= \frac{\beta}{\theta}(1-e^{-\theta T_2})$$

and the equation (7) is obtained.

Proof of eq. (8):

From eq. (7),

$$\frac{\beta}{\theta}(1-e^{-\theta T_2}) = \frac{(r\mu-\beta)}{\theta}(e^{\theta T_1}-1)$$

$$\beta-\beta e^{-\theta T_2} = (-r\mu+\beta)+(r\mu-\beta)e^{\theta T_1}$$

$$-\beta e^{-\theta T_2} = -r\mu+(r\mu-\beta)e^{\theta T_1}$$

Neglecting third and higher order terms in the Taylor series of $e^{\theta T_1}$ and $e^{\theta T_2}$, we have

$$-\beta(1-\theta T_2) = -r\mu + (r\mu - \beta)(1+\theta T_1)$$

$$-\beta + \beta\theta T_2 = -r\mu + r\mu - \beta + (r\mu - \beta)\theta T_1$$

$$\beta T_2 = (r\mu - \beta)T_1$$

$$\frac{T_1}{T_2} = \frac{D}{pr - D} = \frac{\beta}{r\mu - \beta}$$

so $\beta(pr - D) = D(r\mu - \beta)$

$$\beta = \frac{D\mu}{p} \quad \text{or} \quad \mu = \frac{p\beta}{D}$$

and the equation is obtained.

Proof of eq. (9):

The average raw material inventory level can be obtained by integrating I_T over the time interval between time 0 and time (T_1+T_2). Due to the consumption rates, *d(t)*, of raw material over time intervals from time 0 to time T_1 and from time T_1 and time (T_1+T_2) are different, we have to do the integration separately.

(a)The total raw material inventory from time 0 to time T_1:

The raw material consumption rate *d(t)* is ($r\mu$-β) in this period, so eq. (5) can be rewritten as:

$$I_T = I_0 e^{-\theta T} - e^{-\theta T}\int_0^T d(t)e^{\theta t}\,dt = I_0 e^{-\theta T} - \frac{(r\mu - \beta)}{\theta}(1 - e^{-\theta T})$$

Integrating I_T time 0 to time T_1:

$$\int_0^{T_1} I_T\,dT = \int_0^{T_1} I_0 e^{-\theta T}\,dT - \frac{(r\mu - \beta)}{\theta}\int_0^{T_1}(1 - e^{-\theta T})dT$$

Substituting $I_0 = \frac{(r\mu - \beta)}{\theta}(e^{\theta T_1} - 1)$ into the above equation, and neglecting forth and higher order terms in the Taylor series of $e^{\theta T_1}$, we can then obtain

$$\int_0^{T_1} I_T\,dT = (r\mu-\beta)(e^{\theta T_1}-\theta T_1-1)\times\frac{1}{\theta^2}$$

$$\approx (r\mu-\beta)(\frac{T_1^2}{2}+\frac{\theta T_1^3}{6})$$

(b)The total raw material inventory from time T_1 and time (T_1+T_2):

Raw material is depleted at time T_1, and raw material consumption rate *d(t)* is ($-\beta$) in this period. Following the same procedure above, we can obtain

$$\int_0^{T_2} I_T\,dT = \beta(e^{\theta T_2}-\theta T_2-1)\times\frac{1}{\theta^2}$$

$$\approx \beta(\frac{T_2^2}{2}+\frac{\theta T_2^3}{6})$$

The total raw material inventory during a production is ($\int_0^{T_1} I_T\,dT + \int_0^{T_2} I_T\,dT$). Since $T_1=\frac{q}{r}$ and $T_2=\frac{pq}{D}-\frac{q}{r}$, the average raw material inventory level is

$$(\int_0^{T_1} I_T\,dT + \int_0^{T_2} I_T\,dT)/(T_1+T_2)$$

$$=[(r\mu-\beta)(\frac{T_1^2}{2}+\frac{\theta T_1^3}{6})+\beta(\frac{T_2^2}{2}+\frac{\theta T_2^3}{6})]\times\frac{1}{T_1+T_2}$$

$$=\{(r\mu-\beta)(\frac{q}{2r^2}+\frac{\theta q^2}{6r^3})+\beta[\frac{q}{2}(\frac{p}{D}-\frac{1}{r})^2+\frac{\theta q^2}{6}(\frac{p}{D}-\frac{1}{r})^3]\}\times\frac{D}{p}.$$

The total holding cost per unit time for the raw material, then, is

$$MC(\beta,q)=\{\{(r\mu-\beta)(\frac{q}{2r^2}+\frac{\theta q^2}{6r^3})+\beta[\frac{q}{2}(\frac{p}{D}-\frac{1}{r})^2+\frac{\theta q^2}{6}(\frac{p}{D}-\frac{1}{r})^3]\}\times\frac{D}{p}\}\times h_1 c$$

and eq. (9) is obtained.

固定易腐物料供應下製程平均數及生產策略之決定

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摘要

在裝填製程中，製程平均數（目標值）的決定是一個很重要的產品設計問題，尤其當物料成本佔產品成本一大部分時。因為製程平均數決定了製程的良率，進而影響到生產計劃和原物料的採購政策。本文假定一產品中有一原物料（變數）有最低的要求量（下限），如果產品中之原物料含量少於該下限，將被視為不良品，且無任何殘值；產品之製造成本和產品中之原物料的含量成線性關係，且原物料的供應是有限且固定的。又假設此原物料具有易腐性，進而將此易腐性特質加到一個能同時決定製程平均數、生產計劃和原物料的採購政策的模型中。本文將分別針對此種情形建構數學模型，並發展有效演算法來求得最佳解。

關鍵詞彙：存貨模型，採購，易腐性