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Information Role of Liquidity

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ABSTRACT

This study examines the information role of liquidity by first showing that the trail of the two-stage value updating process of an individual security runs from the lagged liquidity return that contains the adverse-selection component to the lead true return. However, this phenomenon could be deceiving, if one thought that the lead true value of an individual security is only updated by the anticipated private information in its own lagged spread. At the portfolio level, we show that the lead true value of a security is actually updated by the anticipated private information in the lagged and contemporaneous spreads of all constituent securities in the portfolio. We call this panorama "the commonality in private information"

JEL classification: G10; G20

Keywords: liquidity, information asymmetry, bid-ask spread

I. INTRODUCTION

The classic view of liquidity is a measure of the convenience of exchanging assets (e.g., Amihud and Meldeson, 1986; Demsetz, 1968; Hirshleifer, 1968). However, the recent literature on market microstructure has seen the role of liquidity become more and more information-oriented. For instances, George, Kaul, and Nimalendran (1991), Glosten and Milgrom (1985), Huang and Stoll (1997), Madhavan, Richardson, and Roomans (1997), and Stoll (1989) discuss the role of private information in the bid-ask spread. This trend may occur because any liquidity related variable, such as the bid-ask spread or trade size, itself can serve as a medium of private information. However, even if the anticipated private information can be correctly formed in these medium variables, the question of how exactly to incorporate private information from the medium variables into security prices remains to be answered. Glosten and Milgrom (1985) propose a model showing that as the number of trades becomes large, a convergence in security values occurs between informed and uninformed traders (see their Proposition 4). This prediction is supported empirically by Jones, Kaul, and Lipson

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(1994) that trade frequency, not trade size, carries information. Their results imply that there must be a lead-lag relation between the anticipated private information and the true value of a security. If the value updating process that incorporates the anticipated private information into a security price follows a time-series fashion and the anticipated private information itself needs to be stored in a medium variable (e.g., the bid-ask spread) before it can be processed, then a trail should exist between this medium variable and the true value of the security. In this study, first we show the existence of this lead-lag relation in the value updating process at the individual security level. The significance of this establishment, often overlooked in the literature[1], is to lay down the foundation for our second set of question: Is it possible for an individual security that the sources of its value updates may also come from the information in the bid-ask spreads of other securities? That is, this trail of a given security might also lead to the value updating of other securities, if private information has some common influence. In the area of "commonality in liquidity", Chordia, Roll, and Subrahmanyam (2000) open a dialogue that liquidity might have a mutual feature in the market. Hasbrouck and Seppi (2001) also find similar empirical evidence even though the influence of their liquidity proxy is relatively small. Furthermore, Coughenour and Saad (2004) study the co-movement of liquidity among a group of securities traded by the same specialist on the New York Stock Exchange. They find that the liquidity of these individual securities co-varies with that of the specialist portfolio after considering market liquidity. Domowitz, Hansch, and Wang (2005) find that the co-movement in liquidity can also result from order types because market orders take liquidity and limit orders provide liquidity. Corwin and Coughenour (2007) find that the amount of attention paid by specialists may also affect liquidity provision in the market. Moulton (2005) find that trade size clustering may be able to explain the variation in the common liquidity measures. Roll, Schwartz, and Subrahmanyam (2007) argue that liquidity facilitates arbitrage in the market and the pricing relation is more efficient when market liquidity is high. Indeed, if the phenomenon of "commonality in liquidity" does exist and the liquidity variables are often served as the medium variables in assessing private information,

[1] Most often in the finance literature the lead-lag relation is related to the predictability of security returns. Grinblatt and Moskowitz (2004) offer a good summary of a variety category of explanations, such as data issues, rational risk-based behavior, and irrational trading behavior.

then could "commonality in liquidity" also result from the common influence of private information? Specifically, if private information contains both firm-specific and market-wide influences, then at the portfolio level it would be interesting to know whether the medium variable, such as the bid-ask spread, affects not only the true value of its own security but also those of others. If so, in term of the processing channels of updating the true values of constituent securities in a portfolio, will private information go through theirs own medium variables or those of others? These are the issues in our second set of question that is silent in the existing literature.

To address these questions, this study first formulates a simple scheme based on the framework in Roll (1984) to decompose the transaction return of individual securities into two elements: the true return and the liquidity return, where the latter contains the adverse-selection component. Furthermore, employing these decomposed return elements, we construct the return covariances as the schematic for the subsequent empirical estimation. Since the liquidity return contains both the adverse selection and the pure liquidity elements, to well explain the flow behavior of private information in the return covariances we need to adopt the model of Glosten (1987) to further separate the adverse-selection component from the liquidity return at the theoretical level. Empirically, at the individual stock level, we expect and find a positive correlation between the liquidity return at time t and the true return at time $t+1$. This positive correlation implies that on average the anticipated private information in the bid-ask spread is correctly perceived and flows from the spread component to the true value of the security through trade direction as predicted by Glosten and Milgrom (1985). This is a two-stage value updating process of private information in the true returns. That is, the anticipated private information is first formed in the spread (i.e., measured in the liquidity return), and then subsequently updates the true value of the security via the trade initiation. The volatility of the adverse-selection component in the spread is the key determinant that how much of the true value of the security is to be updated, while trading noise acts as a camouflage in the value updating process. Next, to explore the common influence in private information, we decompose the observable portfolio return into the portfolio true return and the portfolio liquidity return. We find that there is a more elaborate two-stage value updating process at the portfolio level. The portfolio return serial covariance can be decomposed into four sets of serial covariance components: the lead-lag portfolio true return component covariance, $Cov(R_t^*, R_{t+1}^*)$, the component covariance of the lagged portfolio true return and the lead portfolio liquidity return, $Cov(R_t^*, R_{t+1}^q)$, the component covariance of the lagged portfolio liquidity return and the lead portfolio true return, $Cov(R_t^q, R_{t+1}^*)$ and the lead-lag portfolio liquidity return component covariance, $Cov(R_t^q, R_{t+1}^q)$. In these four sets of portfolio covariance components, we show that unlike at the individual security level[2], some trading noise can be diversified away at the portfolio level. What remains in these decomposed covariance components that perform private information transmission are the cross-security contemporaneous and cross-security lead-lag covariances among the adverse-selection components of the constituent securities in the portfolio. The cross-security contemporaneous covariance of the adverse-selection component is a phenomenon of private information with a common influence shared with other securities contemporaneously (t). This kind of covariance is called the "commonality in private information" of the *first kind*. The cross-security lead-lag covariance of the adverse-selection component involves private information with a common influence but shared with other securities across time, i.e., from time t to $t+1$. Some securities have this piece of private information in their adverse-selection components earlier than other securities, possibly owing to the superior abilities of market makers. Other market makers that lack this particular ability can only gain insights by observing the spread posted by others. This kind of covariance is termed the "commonality in private information" of the *second kind*. The empirical findings presented in this study show that some private information with a common influence is widely anticipated and simultaneously formed in the spread of every security. Other private information with a common influence can only be anticipated by market makers with special abilities. Thus, a lead-lag relationship exists in this private information between the spreads posted by the market makers with and without special insights. Nonetheless, at the

[2] Anand, Chakravarty and Martell (2005) state that "Informed traders are not observable since they take pains to disguise themselves and their trading motives, …" Beneath stealth trading (Alexander and Peterson, 2007; Barclay and Warner, 1993; Chakravarty, 2001), the trail of private information itself involves a complicated structure, which makes detecting private information even harder at individual security level.

portfolio level, despite the existence of both kinds of "commonality in private information", the size of the *first kind* tends to eclipse that of the *second kind*. These findings are helpful in addressing the question posted by Lo and MacKinlay (1990) regarding the economic sources of positive cross-autocorrelation across securities.

The remainder of this paper is organized as follows: Section 2 presents a simple return decomposition model and its applications. Subsequently, section 3 discusses the sample and methodology. Section 4 then presents empirical findings, and followed by concluding remarks in section 5.

II. A SIMPLE MODEL

This section presents a simple scheme based on those described in Roll (1984, Appendix A) and Glosten (1987). The individual security observed return is first divided into two parts, with the first part being the return of the true price that only reflects current public and past revealed private information. The second part is called the liquidity return that contains the anticipated private information and trading noise. The main benefit of these decomposed returns is that it allows us to trace the flow of private information by establishing decomposed covariance measures at the individual security and portfolio levels, making it possible to identify which part of the decomposed covariance contains the flow of private information or trading noises, such as the bid-ask bounce effect[3] (Roll, 1984). This goal, if achieved, will also complement the statement of Glosten (1987, pp. 1294): "*… the entire spread is not the culprit. Rather, the serial correlation, spurious variance, and return bias are due to the portion of the spread arising from inventory costs, monopoly power, clearing costs, etc.*" This study claims that for trading noise the gross-profit component of the spread is not the only source. The adverse-selection component could be another source, when it is across time and across securities in a continuous market setting.

[3] The bid-ask bounce effect in Roll (1984) is that in a continuous market a negative return serial correlation will be observed in the return series of an individual security because of the random occurrence of the transaction price between the ask or bid price, even without the arrival of new information.

1. Decomposition of Individual Return Covariances

In a continuous market, such as the New York Stock Exchange, $Q_{i,t} = +1$ let $p_{i,t}$ be the true price of security i based on the public information set at time t as described in Glosten (1987). In this case, $0 < s_{i,t} < 1$ represents the one-sided percentage spread based on the true price $p_{i,t}$. Furthermore, $Q_{i,t}$ is the trade-initiation variable: $Q_{i,t} = +1$, if it is a buyer-initiated trade, and $Q_{i,t} = -1$, if a seller-initiated trade. Thus, when $Q_{i,t} = +1$, $s_{i,t}$ denotes the ask-sided percentage spread, and when $Q_{i,t} = -1$, $s_{i,t}$ denotes the bid-sided percentage spread. The one-sided percentage spread $s_{i,t}$ can be further decomposed into the one-sided adverse-selection component $0 < s_{i,t}^{AS} < 1$ and the one-sided gross-profit component $0 < s_{i,t}^{GP} < 1$, as described in Glosten (1987). $s_{i,t}^{AS}$ and $s_{i,t}^{GP}$ are assumed to be independent of each other across all securities, i.e., $s_{i,t}^{AS} \perp s_{j,t}^{GP}, \forall i = j, i \neq j, \forall t$, serially and contemporaneously.

This study defines the observable transaction price $P_{i,t}$ of security i at time t as:

$$\begin{aligned} P_{i,t} &= p_{i,t} + p_{i,t} \cdot s_{i,t} \cdot Q_{i,t} \\ &= p_{i,t}(1 + s_{i,t} \cdot Q_{i,t}), \end{aligned} \tag{1}$$

where $s_{i,t} = s_{i,t}^{AS} + s_{i,t}^{GP}$.

The observable transaction return $r_{i,t}$ is defined as:

$$\begin{aligned} r_{i,t} &= \ln P_{i,t} - \ln P_{i,t-1} \\ &= r_{i,t}^{*} + [\ln(1 + s_{i,t} \cdot Q_{i,t}) - \ln(1 + s_{i,t-1} \cdot Q_{i,t-1})] \\ &= r_{i,t}^{*} + \delta_{i,t}, \end{aligned} \tag{2}$$

where $r_{i,t}^{*}$ is the true return and $\delta_{i,t} = \ln(1 + s_{i,t} \cdot Q_{i,t}) - \ln(1 + s_{i,t-1} \cdot Q_{i,t-1})$ is the *liquidity return* from time $t-1$ to t. The liquidity return contains both the changes in the adverse-selection cost and trading noise (e.g., the gross-profit cost).

The innovation process of the true price $p_{i,t+1}$ of security i, at time $t+1$, resembles that described in Glosten (1987, pp. 1300) with some modification of the public information element. That is,

$$\begin{aligned} p_{i,t+1} &= p_{i,t} + p_{i,t} \cdot s^{AS}_{i,t} \cdot Q_{i,t} + p_{i,t} \cdot \varepsilon_{i,t+1} \\ &= p_{i,t} \cdot (1 + s^{AS}_{i,t} \cdot Q_{i,t} + \varepsilon_{i,t+1}), \end{aligned} \tag{3}$$

where $\varepsilon_{i,t+1} \in (-1,+1)$ represents the public information update as the percentage of the true price ($p_{i,t}$) at time t. This study assumes that $\varepsilon_{i,t+1}$ is serially uncorrelated and independent of $s^{AS}_{i,t+k}$ and $s^{GP}_{i,t+k}$, $\forall\, k \in I$.

The continuous compounded return of intrinsic value $r^{*}_{i,t+1}$ (or the true return) from time t to $t+1$ is:

$$\begin{aligned} r^{*}_{i,t+1} &= \ln(\frac{p_{i,t+1}}{p_{i,t}}) = \ln[\frac{p_{i,t} \cdot (1 + s^{AS}_{i,t} \cdot Q_{i,t} + \varepsilon_{i,t+1})}{p_{i,t}}] \\ &= \ln(1 + s^{AS}_{i,t} \cdot Q_{i,t} + \varepsilon_{i,t+1}), \end{aligned} \tag{4}$$

where $s^{AS}_{i,t}$ is the one-sided adverse-selection component of the quote price at time t, $Q_{i,t}$ is the trade direction at time t, and $\varepsilon_{i,t+1}$ is the public information between time t and $t+1$.

The serial covariance of the observable returns of security i from time t to $t+1$ can be decomposed into the following four components:

$$\begin{aligned} & Cov\,(r_{i,t}, r_{i,t+1}) \\ &= Cov\,(r^{*}_{i,t} + \delta_{i,t}, r^{*}_{i,t+1} + \delta_{i,t+1}) \\ &= Cov\,(r^{*}_{i,t} + [\ln(1 + s_{i,t} \cdot Q_{i,t}) - \ln(1 + s_{i,t-1} \cdot Q_{i,t-1})],\ r^{*}_{i,t+1} \\ &\quad + [\ln(1 + s_{i,t+1} \cdot Q_{i,t+1}) - \ln(1 + s_{i,t} \cdot Q_{i,t})]) \\ &= Cov\,(r^{*}_{i,t}, r^{*}_{i,t+1}) + Cov\,[r^{*}_{i,t}, \ln(1 + s_{i,t+1} \cdot Q_{i,t+1}) - \ln(1 + s_{i,t} \cdot Q_{i,t})] \\ &\quad + Cov\,[\ln(1 + s_{i,t} \cdot Q_{i,t}) - \ln(1 + s_{i,t-1} \cdot Q_{i,t-1}), r^{*}_{i,t+1}] \\ &\quad + Cov\,[\ln(1 + s_{i,t} \cdot Q_{i,t}) - \ln(1 + s_{i,t-1} \cdot Q_{i,t-1}), \ln(1 + s_{i,t+1} \cdot Q_{i,t+1}) \\ &\quad - \ln(1 + s_{i,t} \cdot Q_{i,t})], \end{aligned} \tag{5}$$

where $\delta_{i,t} = \ln(1+s_{i,t}\cdot Q_{i,t}) - \ln(1+s_{i,t-1}\cdot Q_{i,t-1})$ is the *liquidity return*.

The decomposition of the observable return autocovariance, $Cov(r_{i,t}, r_{i,t+1})$, in Eq.(5) provides an opportunity to explore the evolution of private information and trading noise (such as, the bid-ask bounce effect), in the market.

(1) The serial covariance of the true return of security i from time t to $t+1$:

$$\begin{aligned}
&Cov(r^{*}_{i,t}, r^{*}_{i,t+1}) \\
&= Cov[\ln(1+s^{AS}_{i,t-1}\cdot Q_{i,t-1}+\varepsilon_{i,t}), \ln(1+s^{AS}_{i,t}\cdot Q_{i,t}+\varepsilon_{i,t+1})] \\
&\approx Cov(s^{AS}_{i,t-1}\cdot Q_{i,t-1}+\varepsilon_{i,t}, s^{AS}_{i,t}\cdot Q_{i,t}+\varepsilon_{i,t+1}) \\
&= Cov(s^{AS}_{i,t-1}\cdot Q_{i,t-1}, s^{AS}_{i,t}\cdot Q_{i,t}) + Cov(s^{AS}_{i,t-1}\cdot Q_{i,t-1}, \varepsilon_{i,t+1}) \\
&\quad + Cov(\varepsilon_{i,t}, s^{AS}_{i,t}\cdot Q_{i,t}) + Cov(\varepsilon_{i,t}, \varepsilon_{i,t+1}) \\
&= Cov(s^{AS}_{i,t-1}\cdot Q_{i,t-1}, s^{AS}_{i,t}\cdot Q_{i,t}) < 0
\end{aligned} \qquad \textbf{(6)}$$

By applying the Delta method, $Cov[\ln(1+s^{AS}_{i,t-1}\cdot Q_{i,t-1}+\varepsilon_{i,t}), \ln(1+s^{AS}_{i,t}\cdot Q_{i,t}+\varepsilon_{i,t+1})]$ can be approximated by $Cov(s^{AS}_{i,t-1}\cdot Q_{i,t-1}+\varepsilon_{i,t}, s^{AS}_{i,t}\cdot Q_{i,t}+\varepsilon_{i,t+1})$ because of the small magnitudes of the one-sided adverse-selection cost $s^{AS}_{i,t}$ and the update of public information $\varepsilon_{i,t}$ (see Appendix A for details). Given $s^{AS}_{i,t}$ and $\varepsilon_{i,t}$ are serially and mutually independent, $Cov(r^{*}_{i,t}, r^{*}_{i,t+1})$ is negative due to the bid-ask bounce effect [4] in $Cov(s^{AS}_{i,t-1}\cdot Q_{i,t-1}, s^{AS}_{i,t}\cdot Q_{i,t})$.

(2) The serial covariance of the return of the true price at time t and the liquidity return at time $t+1$:

[4] By following the framework in Roll (1984), it is forthright to show $Cov(s^{AS}_{i,t-1}\cdot Q_{i,t-1}, s^{AS}_{i,t}\cdot Q_{i,t}) < 0$, whether the spread or its component is random or otherwise.

$$
\begin{aligned}
&Cov(r^*_{i,t}, \delta_{i,t+1}) \\
&= Cov[\ln(1+s^{AS}_{i,t-1}\cdot Q_{i,t-1}+\varepsilon_{i,t}), \ln(1+s_{i,t+1}\cdot Q_{i,t+1})-\ln(1+s_{i,t}\cdot Q_{i,t})] \\
&= Cov[\ln(1+s^{AS}_{i,t-1}\cdot Q_{i,t-1}+\varepsilon_{i,t}), \ln(1+(s^{AS}_{i,t+1}+s^{GP}_{i,t+1})\cdot Q_{i,t+1})] \\
&\quad - Cov[\ln(1+s^{AS}_{i,t-1}\cdot Q_{i,t-1}+\varepsilon_{i,t}), \ln(1+(s^{AS}_{i,t}+s^{GP}_{i,t})\cdot Q_{i,t})] \\
&\approx Cov[s^{AS}_{i,t-1}\cdot Q_{i,t-1}+\varepsilon_{i,t}, (s^{AS}_{i,t+1}+s^{GP}_{i,t+1})\cdot Q_{i,t+1})] \\
&\quad - Cov[s^{AS}_{i,t-1}\cdot Q_{i,t-1}+\varepsilon_{i,t}, (s^{AS}_{i,t}+s^{GP}_{i,t})\cdot Q_{i,t}] \\
&= -Cov[s^{AS}_{i,t-1}\cdot Q_{i,t-1}+\varepsilon_{i,t}, (s^{AS}_{i,t}+s^{GP}_{i,t})\cdot Q_{i,t}] \\
&= -Cov[s^{AS}_{i,t-1}\cdot Q_{i,t-1}, (s^{AS}_{i,t}+s^{GP}_{i,t})\cdot Q_{i,t}] - Cov[\varepsilon_{i,t}, (s^{AS}_{i,t}+s^{GP}_{i,t})\cdot Q_{i,t}] \\
&= -Cov[s^{AS}_{i,t-1}\cdot Q_{i,t-1}, (s^{AS}_{i,t}+s^{GP}_{i,t})\cdot Q_{i,t}] > 0
\end{aligned}
\tag{7}
$$

Again, by applying the Delta method and the assumptions of the adverse-selection cost and the update of public information, this study obtains an approximation that $Cov(r^*_{i,t}, \delta_{i,t+1}) \approx -Cov[s^{AS}_{i,t-1}\cdot Q_{i,t-1}, (s^{AS}_{i,t}+s^{GP}_{i,t})\cdot Q_{i,t}] > 0$ based on the same reason that induces the bid-ask bounce effect.

(3) The serial covariance of the liquidity return at time t and the return of true price at time $t+1$:

$$
\begin{aligned}
&Cov(\delta_{i,t}, r^*_{i,t+1}) \\
&\approx Cov[(s^{AS}_{i,t}+s^{GP}_{i,t})\cdot Q_{i,t}, s^{AS}_{i,t}\cdot Q_{i,t}+\varepsilon_{i,t+1}] \\
&\quad - Cov((s^{AS}_{i,t-1}+s^{GP}_{i,t-1})\cdot Q_{i,t-1}, s^{AS}_{i,t}\cdot Q_{i,t}+\varepsilon_{i,t+1}) \\
&= Var(s^{AS}_{i,t}\cdot Q_{i,t}) + Cov(s^{GP}_{i,t}\cdot Q_{i,t}, s^{AS}_{i,t}\cdot Q_{i,t}) \\
&\quad - [Cov(s^{AS}_{i,t-1}\cdot Q_{i,t-1}, s^{AS}_{i,t}\cdot Q_{i,t}) + Cov(s^{GP}_{i,t-1}\cdot Q_{i,t-1}, s^{AS}_{i,t}\cdot Q_{i,t})] > 0
\end{aligned}
\tag{8}
$$

By applying the Delta method and the assumption of the independence of public information, $\varepsilon_{i,t+1}$, $Cov(\delta_{i,t}, r^*_{i,t+1})$ can be decomposed into two sets of covariance element: $Cov[(s^{AS}_{i,t}+s^{GP}_{i,t})\cdot Q_{i,t}, s^{AS}_{i,t}\cdot Q_{i,t}]$ and $-Cov[(s^{AS}_{i,t-1}+s^{GP}_{i,t-1})\cdot Q_{i,t-1}, s^{AS}_{i,t}\cdot Q_{i,t}]$. The first set of the covariance element $Cov[(s^{AS}_{i,t}+s^{GP}_{i,t})\cdot Q_{i,t}, s^{AS}_{i,t}\cdot Q_{i,t}]$ contains $Var(s^{AS}_{i,t}\cdot Q_{i,t})$ and $Cov(s^{GP}_{i,t}\cdot Q_{i,t}, s^{AS}_{i,t}\cdot Q_{i,t})$, where the latter is positive (see Appendix B). The second set of the covariance element is also positive, again, due to the bid-ask bounce effect. These two sets of the covariance element together make $Cov(\delta_{i,t}, r^*_{i,t+1})$ positive. This two-set covariance decomposition demonstrates that (1) when private information updates the intrinsic value of security i, there is an associated camouflage, i.e., $-[Cov(s_{i,t-1}^{AS}\cdot Q_{i,t-1}, s_{i,t}^{AS}\cdot Q_{i,t}) + Cov(s_{i,t-1}^{GP}\cdot Q_{i,t-1}, s_{i,t}^{AS}\cdot Q_{i,t})] > 0$ in $Cov(\delta_{i,t}, r_{i,t+1}^{*})$, and (2) the volatility of the adverse-selection component $s_{i,t}^{AS}$ (i.e., $Var(s_{i,t}^{AS}\cdot Q_{i,t})$) in the spread is the key determinant of how much private information is to be updated in the security value.

(4) The serial covariance of the liquidity returns of security i between time t and $t+1$:

$$
\begin{aligned}
&Cov(\delta_{i,t}, \delta_{i,t+1})\\
&\approx Cov(s_{i,t}\cdot Q_{i,t}, s_{i,t+1}\cdot Q_{i,t+1}) - Var(s_{i,t}\cdot Q_{i,t})\\
&\quad - Cov(s_{i,t-1}\cdot Q_{i,t-1}, s_{i,t+1}\cdot Q_{i,t+1}) + Cov(s_{i,t-1}\cdot Q_{i,t-1}, s_{i,t}\cdot Q_{i,t})\\
&= Cov(s_{i,t}\cdot Q_{i,t}, s_{i,t+1}\cdot Q_{i,t+1}) - Var(s_{i,t}\cdot Q_{i,t})\\
&\quad + Cov(s_{i,t-1}\cdot Q_{i,t-1}, s_{i,t}\cdot Q_{i,t}) < 0.
\end{aligned}
\tag{9}
$$

Here, $-Cov(s_{i,t-1}\cdot Q_{i,t-1}, s_{i,t+1}\cdot Q_{i,t+1})$ has a theoretical zero covariation because no bid-ask bounce effect exists beyond the lag one and the spread, $s_{i,t}$, or its components are assumed to be mutually and serially independent within security i. Two of the three remaining covariance components in $Cov(\delta_{i,t}, \delta_{i,t+1})$ have negative signs because of the bid-ask bounce effect. Since $s_{i,t} = s_{i,t}^{AS} + s_{i,t}^{GP}$, $-Var(s_{i,t}\cdot Q_{i,t}) < 0$ implies that the sizes of both the adverse-selection and gross-profit components of the spread determine the serial covariance of the liquidity returns, $Cov(\delta_{i,t}, \delta_{i,t+1})$, in addition to the bid-ask bounce effect.

2. Decomposition of Portfolio Return Covariances

Differential patterns of return behavior between individual securities and portfolios are well-known in the literature (e.g., Lo and MacKinlay, 1990). By decomposing portfolio return serial covariance, this study aims to shed light on the trail of private information across securities in the portfolio. Based on the return formation of the individual security in Eq.(4), this study forms portfolio returns and expresses the portfolio serial covariance as follows:

$$
\begin{aligned}
&Cov(R_t, R_{t+1}) \\
&= Cov(R_t^* + R_t^q, R_{t+1}^* + R_{t+1}^q) \\
&= Cov(R_t^*, R_{t+1}^*) + Cov(R_t^*, R_{t+1}^q) + Cov(R_t^q, R_{t+1}^*) + Cov(R_t^q, R_{t+1}^q),
\end{aligned}
\tag{10}
$$

where R_t denotes the portfolio return, R_t^* denotes the portfolio true return of the intrinsic value, and R_t^q denotes the portfolio liquidity return at time t.

These four covariance components are further decomposed with the help of the Delta method (see Appendix A).

2.1 The portfolio serial covariance of the lead and lagged true returns of intrinsic values:

$$
\begin{aligned}
&Cov(R_t^*, R_{t+1}^*) = \sum_{i=1}^{n} Cov(r_{i,t}^*, r_{i,t+1}^*) + \sum_{i=1}^{n} \sum_{j=1, j\neq i}^{n} Cov(r_{i,t}^*, r_{j,t+1}^*) \\
&\approx \sum_{i=1}^{n} Cov(s_{i,t-1}^{AS} \cdot Q_{i,t-1}, s_{i,t}^{AS} \cdot Q_{i,t}) + \sum_{i=1}^{n} \sum_{j=1, j\neq i}^{n} Cov(s_{i,t-1}^{AS} \cdot Q_{i,t-1}, s_{j,t}^{AS} \cdot Q_{j,t}).
\end{aligned}
\tag{11}
$$

$Cov(R_t^*, R_{t+1}^*)$ degenerates into only two terms in Eq.(11) because the public information update, $\varepsilon_{i,t}$, is assumed to be serially and cross-sectionally independent and independent to all private information measures. This study analyzes the signs of these two terms to determine the signs of $\sum_{i=1}^{n} Cov(r_{i,t}^*, r_{i,t+1}^*)$ and $\sum_{i=1}^{n} \sum_{j=1, j\neq i}^{n} Cov(r_{i,t}^*, r_{j,t+1}^*)$, respectively.

(11a) Self-covariances $\sum_{i=1}^{n} \sum_{j=1, j\neq i}^{n} Cov(r_{i,t}^*, r_{i,t+1}^*) \approx \sum_{i=1}^{n} Cov(s_{i,t-1}^{AS} \cdot Q_{i,t-1}, s_{i,t}^{AS} \cdot Q_{i,t})$**:**

Following the assumption $s_{i,t}^{AS} \perp s_{i,t-1}^{AS}, \forall i$, this first term is expected to have a negative sign due to the same reason of the bid-ask bounce effect within individual stocks in Eq.(6).

The individual variances can be diversified away when the number of securities in an equally-weighted portfolio becomes large. If so, with regards to the time series, the self-covariances, such as in (11a), in the portfolio can also be diversified away as the number of securities in the portfolio becomes large. What remains is a set of cross-covariances, such as to be discussed in the followings:

(11b) the cross-covariances $\sum_{i=1}^{n} Cov(r_{i,t}^{*}, r_{j,t+1}^{*}) \approx \sum_{i=1}^{n} \sum_{j=1, j\neq i}^{n} Cov(s_{i,t-1}^{AS} \cdot Q_{i,t-1}, s_{j,t}^{AS} \cdot Q_{j,t})$**:**

The second term in Eq.(11) is a sum of the cross-firm serial covariances, which (i) carries a negative sign, if there are the "cross-firm" lead-lag bid-ask bounce effects that stem from $Q_{i,t-1}$ and $Q_{j,t}$, or (ii) carries a positive sign, if it is part of the "commonality in private information" phenomenon showing the lead-lag cross-inferential private information between the private information update ($s_{i,t-1}^{AS} \cdot Q_{i,t-1}$) in the true value[5] of security i at time t and the private information update ($s_{j,t}^{AS} \cdot Q_{j,t}$) in the true value of security j at time $t+1$.

Appendix C presents a model demonstrating that under a similar market environment to that described in Roll (1984), the "cross-firm" lead-lag covariance of price changes is equal to zero, regardless of whether the probabilities of the transaction price occurred at the ask and bid are symmetrical. That is, if the spreads, s_i and s_j, of securities i and j are constant over time, there will be no "cross-firm" lead-lag bid-ask bounce effect. Consequently, any significance of the covariances in (1b) is likely to come from the properties of the non-constant spreads. In this case, it is worth elaborating the characteristics of the adverse-selection component, $s_{i,t}^{AS}$, of security i at time t. In Glosten and Milgrom (1985), given the possibility of information asymmetry, an anticipation of private information, $s_{i,t}^{AS}$, is formed based on the public information available at time t. This kind of anticipation, $s_{i,t}^{AS}$ and $s_{j,t}^{AS}$, $\forall i \neq j$, could be correlated across securities *contemporaneously* at time t because private information may be anticipated by market makers, just as public information may contain both firm-specific and market-wide information. This kind of common insight in $s_{i,t}^{AS}, \forall i$, is called the "commonality in private information" of the *first kind*. Furthermore, if $s_{i,t}^{AS}, \exists i$,

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[5] It is helpful to review Eqs.(1) to (4) to see how is the private information update ($s_{i,t-1}^{AS} \cdot Q_{i,t-1}$) formed in the true value of security i at time t.

contains another kind of element that is based on not only on public knowledge but also on the special abilities[6] of market makers, then these particular insights, are not correlated across firms contemporaneously but may be correlated across firms in a lead-lag fashion. This phenomenon occurs because this portion of $s_{i,t}^{AS}$ involves the individual capabilities of the market maker and is not revealed until the market maker posts her spread. Others can only adopt it *afterwards* from the posted bid-ask spread. This kind of insight in $s_{i,t}^{AS}, \forall i$, is termed the "commonality in private information" of the *second kind*. Rock (1990) and Seppi (1997) discuss the *second adverse selection problem* that results from the advantageous position of specialists. The orders of uninformed traders could get ahead and achieve trades completion, but actually they could do so is due to the withdraw of the specialist who has insights regarding market conditions. Thus, the first adverse selection problem occurs when uninformed traders face informed traders and the second adverse selection problem occurs when they face the specialist with insights. The "commonality in private information" of the *second kind* complements the view of Rock (1990) and Seppi (1997) regarding the insights of market makers against those of uninformed traders.

In the empirical section, this study estimates $Cov(R_t^*, R_{t+1}^*)$ and its two components, $\sum_{i=1}^{n} Cov(s_{i,t-1}^{AS} \cdot Q_{i,t-1}, s_{i,t}^{AS} \cdot Q_{i,t}) < 0$ (to verify the hypothesis of the bid-ask bounce effect for individual securities) and $\sum_{i=1}^{n} \sum_{j=1, j \neq i}^{n} Cov(s_{i,t-1}^{AS} \cdot Q_{i,t-1}, s_{j,t}^{AS} \cdot Q_{j,t}) > 0$ (to verify the hypothesis of the "commonality in private information" of the *second kind*.), separately.

2.2 The portfolio serial covariance of the lagged true return and lead liquidity return:

[6] Each market maker may have his/her own unique ability to interpret public information and form his/her adverse-selection component that at least part of it is inimitably insightful. For the discussion regarding the ability of uninformed traders, such as market makers, can be seen in, e.g., Bloomfield, O'Hara (1999), Brunnermeier (2005), Chordia, Sarkar, and Subrahmanyam (2005) and Naik, Neuberger, Viswanathan (1999).

$$Cov(R_t^*, R_{t+1}^q) = \sum_{i=1}^{n} Cov(r_{i,t}^*, \delta_{i,t+1}) + \sum_{i=1}^{n} \sum_{j=1, j \neq i}^{n} Cov(r_{i,t}^*, \delta_{j,t+1})$$

$$\approx \sum_{i=1}^{n} -[Cov(s_{i,t-1}^{AS} \cdot Q_{i,t-1}, s_{i,t}^{AS} \cdot Q_{i,t}) + Cov(s_{i,t-1}^{AS} \cdot Q_{i,t-1}, s_{i,t}^{GP} \cdot Q_{i,t})] \quad \textbf{(12)}$$

$$+ \sum_{i=1}^{n} \sum_{j=1, j \neq i}^{n} -[Cov(s_{i,t-1}^{AS} \cdot Q_{i,t-1}, s_{j,t}^{AS} \cdot Q_{j,t}) + Cov(s_{i,t-1}^{AS} \cdot Q_{i,t-1}, s_{j,t}^{GP} \cdot Q_{j,t})]$$

By the model assumptions, $Cov(R_t^*, R_{t+1}^q)$ degenerates into two parts in Eq.(12). The first part contains two components of self-covariance while the second part contains two components of cross-firm covariance.

(12a) Components of self-covariance:

$$\sum_{i=1}^{n} Cov(r_{i,t}^*, \delta_{i,t+1}) \approx -\sum_{i=1}^{n} Cov(s_{i,t-1}^{AS} \cdot Q_{i,t-1}, s_{i,t}^{AS} \cdot Q_{i,t}) - \sum_{i=1}^{n} Cov(s_{i,t-1}^{AS} \cdot Q_{i,t-1}, s_{i,t}^{GP} \cdot Q_{i,t}).$$

Following the assumption $s_{i,t}^{AS} \perp s_{i,t-1}^{AS}, \forall i$ and $s_{i,t}^{AS} \perp s_{i,t\pm1}^{GP}, \forall i$ again, due to the bid-ask bounce effect and the existing negative sign at the front of them, this study predicts that both component covariances in the first part of Eq.(12) are positive, as occurs in Eq.(7). That is, $\sum_{i=1}^{n} Cov(r_{i,t}^*, \delta_{i,t+1}) > 0$.

(12b) Components of cross-firm covariances:

$$\sum_{i=1}^{n} \sum_{j=1, j \neq i}^{n} Cov\ (r_{i,t}^*, \delta_{j,t+1})$$

$$= \sum_{i=1}^{n} \sum_{j=1, j \neq i}^{n} -[Cov\ (s_{i,t-1}^{AS} \cdot Q_{i,t-1}, s_{j,t}^{AS} \cdot Q_{j,t}) + Cov\ (s_{i,t-1}^{AS} \cdot Q_{i,t-1}, s_{j,t}^{GP} \cdot Q_{j,t})]$$

First, $Cov(s_{i,t-1}^{AS} \cdot Q_{i,t-1}, s_{j,t}^{GP} \cdot Q_{j,t}) = 0$ due to the hypothesis regarding the cross-firm lead-lag bid-ask bounce effect (Appendix C) and the assumption of $s_{i,t}^{AS} \perp s_{i,t\pm1}^{GP}, \forall i$. If $\sum_{i=1}^{n} \sum_{j=1, j \neq i}^{n} Cov(s_{i,t-1}^{AS} \cdot Q_{i,t-1}, s_{j,t}^{AS} \cdot Q_{j,t}) > 0$, it is consistent to the hypothesis of the "commonality in private information" of the *second kind*. Thus, we should observe $\sum_{i=1}^{n} \sum_{j=1, j \neq i}^{n} Cov(r_{i,t}^*, \delta_{j,t+1}) \approx \sum_{i=1}^{n} \sum_{j=1, j \neq i}^{n} - Cov(s_{i,t-1}^{AS} \cdot Q_{i,t-1}, s_{j,t}^{AS} \cdot Q_{j,t}) < 0.$ The empirical results in (12a) and (12b) need to be consistent with those in (11a) and (11b), respectively, because they are based on the same hypotheses of the individual security bid-ask bounce effect and the "commonality in private information" of the *second kind*, respectively.

2.3 The portfolio covariance of the lagged liquidity return and the lead true return:

$$
\begin{aligned}
Cov(R_t^q, R_{t+1}^*) &= \sum_{i=1}^{n} Cov(\delta_{i,t}, r_{i,t+1}^*) + \sum_{i=1}^{n}\sum_{j=1, j\neq i}^{n} Cov(\delta_{i,t}, r_{j,t+1}^*) \\
&\approx \sum_{i=1}^{n}[Var(s_{i,t}^{AS}\cdot Q_{i,t}) + Cov(s_{i,t}^{GP}\cdot Q_{i,t}, s_{i,t}^{AS}\cdot Q_{i,t})] \\
&\quad + \sum_{i=1}^{n} -[Cov(s_{i,t-1}^{AS}\cdot Q_{i,t-1}, s_{i,t}^{AS}\cdot Q_{i,t}) + Cov(s_{i,t-1}^{GP}\cdot Q_{i,t-1}, s_{i,t}^{AS}\cdot Q_{i,t})] \qquad \textbf{(13)} \\
&\quad + \sum_{i=1}^{n}\sum_{j=1, j\neq i}^{n}[Cov(s_{i,t}^{AS}\cdot Q_{i,t}, s_{j,t}^{AS}\cdot Q_{j,t}) + Cov(s_{i,t}^{GP}\cdot Q_{i,t}, s_{j,t}^{AS}\cdot Q_{j,t})] \\
&\quad + \sum_{i=1}^{n}\sum_{j=1, j\neq i}^{n} -[Cov(s_{i,t-1}^{AS}\cdot Q_{i,t-1}, s_{j,t}^{AS}\cdot Q_{j,t}) + Cov(s_{i,t-1}^{GP}\cdot Q_{i,t-1}, s_{j,t}^{AS}\cdot Q_{j,t})].
\end{aligned}
$$

By the model assumptions, $Cov(R_t^q, R_{t+1}^*)$ degenerates into two parts in Eq.(13). The first part, $\sum_{i=1}^{n} Cov(\delta_{i,t}, r_{i,t+1}^*)$, contains four components of self-covariances and the second part, $\sum_{i=1}^{n}\sum_{j=1, j\neq i}^{n} Cov(\delta_{i,t}, r_{j,t+1}^*)$, contains four components of cross-firm covariances.

(13a) Components of self-covariances:

$\sum_{i=1}^{n} Var(s_{i,t}^{AS}\cdot Q_{i,t}) > 0$ in $\sum_{i=1}^{n} Cov(\delta_{i,t}, r_{i,t+1}^*)$ is the key element showing that at time t the anticipated private information in $\delta_{i,t}$ has been transferred to the true return, $r_{i,t+1}^*$, at time $t+1$ (see Appendix D for the detail). Furthermore, from Appendix B, $\sum_{i=1}^{n} Cov(s_{i,t}^{GP}\cdot Q_{i,t}, s_{i,t}^{AS}\cdot Q_{i,t})$ should be positive. Both $Cov(s_{i,t-1}^{AS}\cdot Q_{i,t-1}, s_{i,t}^{AS}\cdot Q_{i,t})$ and $Cov(s_{i,t-1}^{GP}\cdot Q_{i,t-1}, s_{i,t}^{AS}\cdot Q_{i,t})$ are negative due to the bid-ask bounce effect. However, they also have a negative sign at the front of them. Thus, the sum of the self-covariances in Eq.(13), i.e., $\sum_{i=1}^{n} Cov(\delta_{i,t}, r^{*}_{i,t+1})$, should be positive to reflect the bid-ask bounce effect and the transfer of private information, as in Eq.(8).

(13b) Components of cross-firm covariances:

Both $\sum_{i=1}^{n}\sum_{j=1, j\neq i}^{n} Cov(s^{GP}_{i,t}\cdot Q_{i,t}, s^{AS}_{j,t}\cdot Q_{j,t})$ and $\sum_{i=1}^{n}\sum_{j=1, j\neq i}^{n} - Cov(s^{GP}_{i,t-1}\cdot Q_{i,t-1}, s^{AS}_{j,t}\cdot Q_{j,t})$ are expected to equal zero because of the assumptions $s^{AS}_{i,t} \perp s^{GP}_{j,t\pm1}, \forall i, j, i\neq j$ and the independence of contemporaneous and cross-serial trade direction among securities, i.e., $Q_{i,t-1} \perp Q_{j,t}, \forall i, j, i\neq j$. In short, this study predicts that these two sets of covariances equal zero because of none existence of the "cross-firm" bid-ask bounce effect [7], given the above assumptions (Appendix C). $\sum_{i=1}^{n}\sum_{j=1, j\neq i}^{n} Cov(s^{AS}_{i,t}\cdot Q_{i,t}, s^{AS}_{j,t}\cdot Q_{j,t})$ is expected to be positive due to the hypothesis of "commonality in private information" of the *first kind*, i.e., private information contemporaneously spills over among securities. The study also expects $\sum_{i=1}^{n}\sum_{j=1, j\neq i}^{n} - Cov(s^{AS}_{i,t-1}\cdot Q_{i,t-1}, s^{AS}_{j,t}\cdot Q_{j,t})$ to be positive due to the hypothesis of "commonality in private information" of the *second kind*, i.e., the private information cross-security lead-lag spills over among securities. Nonetheless, $Cov(s^{AS}_{i,t-1}\cdot Q_{i,t-1}, s^{AS}_{j,t}\cdot Q_{j,t})$ already has a negative sign in front of it. Thus, the sign of the sum of the four cross-firm covariance components can only be found out empirically.

In the empirical section, this study is going to estimate the sum of the self-covariance, $\sum_{i=1}^{n} Cov(\delta_{i,t1}, r^{*}_{i,t+1})$ to verify its consistency with the hypotheses of the bid-ask bounce effect, and to estimate the sum of the cross-firm covariances,

[7] Understand that $\sum_{i=1}^{n}\sum_{j=1, j\neq i}^{n} Cov(s^{GP}_{i,t}\cdot Q_{i,t}, s^{AS}_{j,t}\cdot Q_{j,t})$ is a contemporaneous cross-firm covariance and it is not a lead-lag cross-firm covariance as described in Appendix C. However, it is straight-forward to show that the same result can be concluded.

$\sum_{i=1}^{n}\sum_{j=1, j\neq i}^{n} Cov(\delta_{i,t}, r_{j,t+1}^{*})$ to verify the hypotheses of the "commonality in private information" of the *first* and *second kind*. Particular, if the "commonality in private information" of the *first kind* is stronger than that of the *second kind*, then we should expect $\sum_{i=1}^{n}\sum_{j=1, j\neq i}^{n} Cov(\delta_{i,t}, r_{j,t+1}^{*}) > 0$, and vice versa.

2.4 The portfolio covariance of the lead-lag liquidity returns:

$$\begin{aligned}
Cov(R_t^q, R_{t+1}^q) &= \sum_{i=1}^{n} Cov(\delta_{i,t}, \delta_{i,t+1}) + \sum_{i=1}^{n}\sum_{j=1, j\neq i}^{n} Cov(\delta_{i,t}, \delta_{j,t+1}) \\
&\approx \sum_{i=1}^{n} Cov(s_{i,t}\cdot Q_{i,t}, s_{i,t+1}\cdot Q_{i,t+1}) - \sum_{i=1}^{n} Var(s_{i,t}\cdot Q_{i,t}) \\
&\quad + \sum_{i=1}^{n} Cov(s_{i,t-1}\cdot Q_{i,t-1}, s_{i,t}\cdot Q_{i,t}) + \sum_{i=1}^{n}\sum_{j=1, j\neq i}^{n} Cov(s_{i,t}^{AS}\cdot Q_{i,t}, s_{j,t+1}^{AS}\cdot Q_{j,t+1}) \\
&\quad - \sum_{i=1}^{n}\sum_{j=1, j\neq i}^{n} Cov(s_{i,t}^{AS}\cdot Q_{i,t}, s_{j,t}^{AS}\cdot Q_{j,t}) + \sum_{i=1}^{n}\sum_{j=1, j\neq i}^{n} Cov(s_{i,t-1}^{AS}\cdot Q_{i,t-1}, s_{j,t}^{AS}\cdot Q_{j,t}).
\end{aligned} \tag{14}$$

By the model assumptions aforementioned, $Cov(R_t^q, R_{t+1}^q)$ can be degenerated into two parts, including a set of self-covariance components and a set of cross-firm covariance components. The self-covariance of the liquidity return of an individual security is conventionally considered to simply comprise trading noise. Interestingly, at the portfolio level the self-covariance of the liquidity return becomes the resonance of private information.

(14a) Self-covariance components of $\sum_{i=1}^{n} Cov(\delta_{i,t}, \delta_{i,t+1})$:

In the model assumptions, the spread components, $s_{i,t}^{AS}$, $s_{i,t}^{GP}$, and the trade direction, $Q_{i,t}$ are mutually and serially independent. Thus, $\sum_{i=1}^{n} Cov(\delta_{i,t}, \delta_{i,t+1})$ represents the impact of trading noises, enhanced by the size of the adverse-selection component, $s_{i,t}^{AS}$. All three covariance components of $\sum_{i=1}^{n} Cov(\delta_{i,t}, \delta_{i,t+1})$ are expected to be negative, particularly owing to the bid-ask bounce effect in the first and the third covariance components, as in Eq.(9).

(14b) The cross-firm covariance components of $\sum_{i=1}^{n}\sum_{j=1,j\neq i}^{n}Cov(\delta_{i,t},\delta_{j,t+1})$**:**

By the model assumptions, $s_{i,t}^{AS}\perp s_{j,t\pm1}^{GP}$ and $s_{i,t}^{GP}\perp s_{j,t\pm1}^{GP},\forall i,j,\forall t$, we have degenerated $\sum_{i=1}^{n}\sum_{j=1,j\neq i}^{n}Cov(\delta_{i,t},\delta_{j,t+1})$ into three sets of cross-firm covariances in Eq.(14). Appendix C demonstrates that the 'cross-firm' lead-lag bid-ask bounce effect does not exist, and thus $Cov(s_{i,t}^{AS}\cdot Q_{i,t},s_{j,t}^{AS}\cdot Q_{j,t})$ is expected to be positive due to the "commonality in private information" of the *first kind* and $Cov(s_{i,t}^{AS}\cdot Q_{i,t},s_{j,t+1}^{AS}\cdot Q_{j,t+1})$ and $Cov(s_{i,t-1}^{AS}\cdot Q_{i,t-1},s_{j,t}^{AS}\cdot Q_{j,t})$ are expected to be positive due to the "commonality in private information" of the *second kind*. However, the sign of $Cov(R_{i,t}^{q},R_{j,t+1}^{q})$ is subject to empirical result because $-\sum_{i=1}^{n}\sum_{j=1,j\neq i}^{n}Cov(s_{i,t}^{AS}\cdot Q_{i,t},s_{j,t}^{AS}\cdot Q_{j,t})$ is negative.

In the empirical section, we will estimate both $\sum_{i=1}^{n}Cov(\delta_{i,t},\delta_{i,t+1})$ and $\sum_{i=1}^{n}\sum_{j=1,j\neq i}^{n}Cov(\delta_{i,t},\delta_{j,t+1})$ to verify the hypotheses of the bid-ask bounce effect and the "commonality in private information" of the *first* and *second kind*, respectively. Particularly, if the "commonality in private information" of the *first kind* is stronger than that of the *second kind*, then we should expect $\sum_{i=1}^{n}\sum_{j=1,j\neq i}^{n}Cov(\delta_{i,t},\delta_{j,t+1})<0$, and vice versa. That is, the empirical results on (14b) need to be consistent with those on (13b).

III. DATA AND METHODOLOGY

1. Data and Data Configuration

The data of this study are obtained from the New York Stock Exchange, Trade and Quote database and the sample period runs from January to December 2003. NYSE TAQ data contain two separate files, namely trade file and quote file, this study first employs Lee and Ready (1991) algorithm to merge trade and quote information. For the purpose of this study, we do not consider market opening trades, which are under the call market trading mechanism. The sample only includes NYSE-listed common stocks, which have daily mean prices ranging between one dollar and five hundred dollars. For the preparation of various trading interval portfolios, for instance the five-min intraday trading interval, we select the price observation at the beginning of each five-min intraday trading interval and that at the end of the last five-min trading interval. The last observation of the last five-min intraday trading interval cannot be the same as the first price observation. The five-min case here contains a total of 78 return observations per trading day. The sample period contains 248 trading days. On any given trading day, we select stocks for which full trading interval return observations are available. We form daily five-min, ten-min, 13-min, 15-min, 26-min, and 30-min intraday equally-weighted[8] portfolios. Portfolios with different trading interval lengths can help show the robustness of our results and reduce the analysis bias due to the nonsynchronous trading.

In order to match the attributes of the variables in our sample with the characteristics of our model, we further assume that the ask spread (the distance between the ask price and the true value of the security) and bid spread (the distance between the true value of the security and the bid price) are symmetric (e.g., Avramov, Chordia, and Goyal, 2006; Comerton-Forde and Rydge, 2004; Goldstein and Kavajecz, 2004; Huang and Stoll, 1997; Naes and Skjeltorp, 2004; Roll, 1984). Thus, the mid-point price, $p_{i,t}^{M}$, the average of the ask and the bid, is a proxy for the true value of security i, $p_{i,t}^{*}$. $r_{i,t}^{M}$ denotes the change of the mid-point prices between time t and $t-1$ and is a proxy for the true return ($r_{i,t}^{*}$) of the true value of security i. We address the potential bias induced by using the mid-point price (return) to proxy the true value (return) of security in Appendix E. Finally, the observable return, $r_{i,t}$, is the change of the transaction price of security i

[8] A value-weighted portfolio requires a large number of securities in order to have a reasonable proxy for the total market value and thus the value-weights for the constituent securities. However, in our sample (or any sample) it is less likely to acquire a large number of constituent securities that they all trade in each of the n-min trading interval of a trading day. What we need is just a group of securities that carries enough liquidity and information during a trading day whether they are larger or smaller firms. Here, an equally-weighted portfolio is more suitable for our purpose because it is not the value weights but the flow of information is what it counts.

between times t and $t-1$. Thus, the difference between the observable return, $r_{i,t}$, and the mid-point return, $r_{i,t}^{M}$, is the proxy for the liquidity return, $\delta_{i,t}$, in Eq.(2). Table 1 lists the summary statistics of the sample. For each security, we first calculates its daily means of price, market capitalization, transactional returns, mid-point returns, liquidity returns, quoted percentage and dollar spreads, effective percentage and dollar spreads (Petersen and Fialkowski, 1994) and daily Roll spread (Roll, 1984). Next, we compute the cross-sectional means of these daily means (or Roll spread) of interest across the entire sample period. On average, the transactional return is positive, as are its components, i.e., the mid-point return and the liquidity return. Larger magnitude of the mid-point return than that of the liquidity return implies that the influence of public information overshadows that of private information. As expected, the size of the quoted spread is greater than that of the effective spread (e.g., Lee, Mucklow, and Ready, 1993; Petersen and Fialkowski, 1994; Roll, 1984). The number of (daily) securities in the computation of the Roll spread is 545,443 and thus is less than these of the other variables (558,391). The difference occurs because the daily transactional return covariances of some securities are positive, and thus are unsuitable for taking a square root (Roll, 1984).

Table 1. Summary statistics

statistics / variables	No. security	Mean	Standard deviation	Minimum	Maximum
Price	558391	24.0934	18.9190	1.00	460.0000
MktCap (millions)	558391	4522.7199	17436.8640	0.4687	356775.0500
Transactional return	558391	9.7191E-6	0.000458	-0.0213	0.0184
Mid-point return	558391	7.4086E-6	0.000447	-0.0272	0.0172
Liquidity return	558391	2.3028E-6	0.000268	-0.0131	0.0139
Quoted % spread	558391	0.0033	0.004497	0.0001	0.1355
Quoted $ spread	558391	0.0467	0.052188	0.0100	2.6654
Effective % spread	558391	0.0026	0.003565	0.0001	0.1043
Effective $ spread	558391	0.0371	0.042288	0.0043	2.3688
Roll spread	545443	0.0020	0.002706	3.0736E-7	0.1428

For each security, we first compute its daily means of price, market capitalization, transactional return, mid-point return (a proxy for the true return), liquidity return, quoted percentage and dollar spreads, the effective percentage and dollar spreads (Petersen and Fialkowski, 1994) and the daily Roll spread (Roll, 1984). We then compute the cross-sectional means of these daily means (or Roll spread) in interest across the entire sample period. We employ the NYSE TAQ data and only select firms listed on the NYSE and only consider firms with daily mean prices greater than $1 and less than $500. The sample period is from January to December 2003.

IV. EMPIRICAL ANALYSIS

1. Individual Security Covariance and Its Covariance Components

In the sample, for each security, we first compute the daily transactional serial covariance $Cov(r_{i,t}, r_{i,t+1})$ and its four covariance components: the serial mid-point return covariance $Cov(r_{i,t}^M, r_{i,t+1}^M)$ to proxy $Cov(r_{i,t}^*, r_{i,t+1}^*)$ in Eq.(6), the lagged mid-point return and lead liquidity return covariance $Cov(r_{i,t}^M, \delta_{i,t+1}^M)$ to proxy $Cov(r_{i,t}^*, \delta_{i,t+1})$ in Eq.(7), the lagged liquidity return and lead mid-point return covariance $Cov(\delta_{i,t}^M, r_{i,t+1}^M)$ to proxy $Cov(\delta_{i,t}, r_{i,t+1}^*)$ in Eq.(8), and the serial liquidity return covariance $Cov(\delta_{i,t}^M, \delta_{i,t+1}^M)$ to proxy $Cov(\delta_{i,t}, \delta_{i,t+1})$ in Eq.(9), where $r_{i,t}^M$ is the mid-point return to proxy the true return $r_{i,t}^*$ and $\delta_{i,t}^M$ is the empirical liquidity return to proxy the (theoretical) liquidity return $\delta_{i,t}$ at time t by assuming the bid-ask spread symmetry. In the brackets, we also report their corresponding serial correlations. We then compute the cross-sectional means of these daily covariances and their related statistics across the 12-month sample period.

Table 2. The statistics of individual security's transactional serial covariance and its decomposition

covariance \ statistics	No. security	Mean	Standard deviation	Minimum	Maximum
$Cov(r_{i,t}, r_{i,t+1})$	558391	-6.684835E-7* [-0.086835*]	0.000010 [0.158425]	-0.002936 [-0.999993]	0.001286 [0.999999]
$Cov(r^M_{i,t}, r^M_{i,t+1})$	558391	-8.881233E-7* [-0.074158*]	0.000015 [0.147378]	-0.005636 [-1.000000]	0.000987 [0.995124]
$Cov(r^M_{i,t}, r^M_{i,t+1}) \geq 0$	162872	6.629560E-7* [0.0729005*]	6.787633E-6 [0.0847934]	0 [0]	0.000987 [0.995124]
$Cov(r^M_{i,t}, r^M_{i,t+1}) < 0$	395519	-1.526847E-6* [-0.134680*]	0.000017 [0.123033]	-0.0056363 [-1.0000000]	-6.0585E-26 [-4.284E-19]
$Cov(r^M_{i,t}, \delta^M_{i,t+1})$	558391	7.152350E-7* [0.0767619*]	0.000015 [0.154114]	-0.001126 [-0.993933]	0.004918 [0.986334]
$Cov(r^M_{i,t}, \delta^M_{i,t+1}) \geq 0$	434149	1.404042E-6* [0.129069*]	0.000016 [0.114336]	0 [0]	0.004918 [0.986334]
$Cov(r^M_{i,t}, \delta^M_{i,t+1}) < 0$	124242	-1.691718E-6* [-0.105994*]	0.000013 [0.134540]	-0.001126 [-0.993933]	-6.4624E-25 [-1.189E-18]
$Cov(\delta^M_{i,t}, r^M_{i,t+1})$	558391	2.166944E-6* [0.254794*]	0.000020 [0.137382]	-0.000472 [-0.993551]	0.005951 [0.996696]
$Cov(\delta^M_{i,t}, r^M_{i,t+1}) > 0$	542412	2.301541E-6* [0.266673*]	0.000020 [0.117184]	0 [0]	0.005951 [0.996696]
$Cov(\delta^M_{i,t}, r^M_{i,t+1}) \leq 0$	15979	-2.402006E-6* [-0.148536*]	0.000011 [0.160991]	-0.000472 [-0.993551]	-1.3846E-19 [-9.154E-14]
$Cov(\delta^M_{i,t}, \delta^M_{i,t+1})$	558391	-2.662539E-6* [-0.2937978*]	0.000021 [0.124382]	-0.005099 [-0.998316]	0.000931 [0.986073]
$Cov(\delta^M_{i,t}, \delta^M_{i,t+1}) \geq 0$	12948	3.509898E-6* [0.1253584*]	0.000018 [0.129837]	0 [0]	0.000931 [0.986073]
$Cov(\delta^M_{i,t}, \delta^M_{i,t+1}) < 0$	545443	-2.809064E-6* [-0.303746*]	0.000021 [0.105680]	-0.005099 [-0.998316]	-2.362E-14 [-1.0817E-7]

* t-test, significantly different from zero at 1% level or higher.

For each security, we first compute its daily transactional serial covariance $Cov(r_{i,t}, r_{i,t+1})$ and its four component covariances, i.e., the serial mid-point return covariance $Cov(r^M_{i,t}, r^M_{i,t+1})$ to proxy $Cov(r^*_{i,t}, r^*_{i,t+1})$, the lagged mid-point return and lead liquidity return covariance $Cov(r^M_{i,t}, \delta^M_{i,t+1})$ to proxy $Cov(r^*_{i,t}, \delta_{i,t+1})$, the lagged liquidity return and lead mid-point return covariance $Cov(\delta^M_{i,t}, r^M_{i,t+1})$ to proxy $Cov(\delta_{i,t}, r^*_{i,t+1})$, and the serial liquidity return covariance $Cov(\delta^M_{i,t}, \delta^M_{i,t+1})$ is to proxy $Cov(\delta_{i,t}, \delta_{i,t+1})$, where $r^M_{i,t}$ is the mid-point return and $\delta^M_{i,t}$ is the empirical liquidity return at time t. In the brackets, we also report their corresponding serial correlations. We then across the twelve-month sample period compute the cross-sectional means of these daily covariances and their related statistics. We employ the NYSE TAQ data and only select firms listed on the NYSE and only consider firms with daily mean prices greater than \$1 and less than \$500. The sample period is from January to December 2003.

In Table 2, the cross-sectional mean of the lead-lag mid-point return covariance $Cov(r^M_{i,t}, r^M_{i,t+1})$ is negative that is consistent to our prediction in Eq.(6) to reflect the bid-ask bounce effect. The cross-sectional mean of the lagged mid-point return and lead liquidity return covariance $Cov(r^M_{i,t}, \delta^M_{i,t+1})$ is positive, because it already has a negative sign in front of it (see Eq.(7)), to reflect the bid-ask bounce effect (Roll, 1984). The cross-sectional mean of the lagged liquidity return and lead mid-point return covariance $Cov(\delta^M_{i,t}, r^M_{i,t+1})$ is positive to reflect the volatility in the private information element, $s^{AS}_{i,t}$, at time t camouflaged by the bid-ask bounce effect in Eq.(8). $Cov(\delta^M_{i,t}, \delta^M_{i,t+1})$ is negative as predicted in Eq.(9) to reflect trading noise, such as the bid-ask bounce effect and $-Var(s_{i,t}Q_{i,t})$. In these findings, we notice that the adverse-selection component, $s^{AS}_{i,t}$, a conventional private information variable, actually plays a more dynamic role in the spread structure across time. Informed traders tend to trade when market liquidity is high and liquidity traders prefer to trade in a market that is informative[9]. When facing information asymmetry, the uninformed market maker has to estimate the size of private information to form $s^{AS}_{i,t}$ that will

[9] For instances, Admati and Pfleiderer (1988), Holden and Subrahmanyam (1992), and Kyle (1985). Furthermore, Anand, Chakravarty and Martell (2005) state that “[i]nformed traders are not observable since they take pains to disguise themselves and their trading motives, …”

inevitably interact with other components evolved in the spread and therefore create additional trading noise. Glosten (1987, Proposition 1) claims that the larger the size of the gross-profit component in the spread the larger the size of the adverse-selection component. Our results complement to his view: The larger the size of the adverse-selection component the larger the size of trading noise in the transaction return covariance (see Eq.(5)). This phenomenon is termed the *dilemma of adverse selection*.

2. Randomness of Spread and Trade Direction

By the model assumptions, the spread components are random but independent, i.e., $s_{i,t}^{AS} \perp s_{i,t-1}^{AS}$ and $s_{i,t}^{AS} \perp s_{i,t\pm1}^{GP}$. In this section, we examine the effects of the random spread regarding the covariation in Eq.(5), in addition to the bid-ask bounce effect caused by the change in the trade direction. In Table 3, we regress daily-firm $Cov(r_{i,t}^{M}, r_{i,t+1}^{M})$, $Cov(r_{i,t}^{M}, \delta_{i,t+1}^{M})$, $Cov(\delta_{i,t}^{M}, r_{i,t+1}^{M})$, $Cov(\delta_{i,t}^{M}, \delta_{i,t+1}^{M})$, and *Roll* spread, respectively, on three explanatory variables. The first explanatory variable is the daily mean market capitalization, $MktCap_i$, which serves as the control variable for firm's characteristics. The second explanatory variable is the daily asymmetric frequency ratio of the trade-direction continuation frequency over the trade-direction discontinuation frequency at the natural logarithm level, $AsyFreqRatio_i$, to control for the bid-ask bounce effect (Roll, 1984). Jones, Kaul, and Lipson (1994) claim that instead of trade size, it is trade frequency that carries information. Their argument is based on the theoretical models of Easley and O'Hara (1992) and Harris and Raviv (1993) that trade frequency is a determinant of the asset prices. Nonetheless, our $AsyFreqRatio_i$ goes beyond the scope of trade frequency, it is a proportion measure of the continuation over discontinuation of trade direction. The trade-direction discontinuation implies a change of trade direction, which will induce the bid-ask bounce effect. The third explanatory variable is the variance of the effective percentage spread, $VarSpread_i$ (Petersen and Fialkowski, 1994), which is a proxy for the informativeness of private information, contrasting to a constant bid-ask spread. We expect that the effect of $VarSpread_i$ on these four covariance components is opposite to that of $MktCap_i$. That is, for smaller firms, the flow of private information is much more uncertain than that of larger firms, which are followed by more analysts in the market (Bhushan, 1989). The basic model of the regressions is as follows:

$$Dependent_i = \alpha_0 + \alpha_1 \cdot MktCap_i + \alpha_2 \cdot VarSpread_i + \alpha_3 \cdot AsyFreqRatio_i + \varepsilon_i, \quad \textbf{(15)}$$

where ε_i is the random error term.

$MktCap_i$ has statistically significant negative impact on the dependent variables, i.e., Roll spread and $Cov(\delta^M_{i,t}, r^M_{i,t+1})$, because larger firms tend to have smaller spreads. Furthermore, it has a statistically significant positive impact on $Cov(\delta^M_{i,t}, \delta^M_{i,t+1})$, which implies that the magnitude of the negative $Cov(\delta^M_{i,t}, \delta^M_{i,t+1})$ is smaller for larger firms. The daily asymmetric frequency ratio, $AsyFreqRatio_i$, should have a similar effect on these four component covariances as $MktCap_i$. The higher the relative trade-direction continuation frequency should mitigate the size of the bid-ask bounce effect. Thus, $AsyFreqRatio_i$ has a statistically significant negative impact on Roll's spread, $Cov(r^M_{i,t}, \delta^M_{i,t+1})$, and $Cov(\delta^M_{i,t}, r^M_{i,t+1})$, but a statistically significant positive impact on $Cov(r^M_{i,t}, r^M_{i,t+1})$ and $Cov(\delta^M_{i,t}, \delta^M_{i,t+1})$. As expected, the variance of the effective percentage spread, $VarSpread_i$, reinforces the magnitudes of these four covariance components and the Roll's spread to reflect the randomness of the spread, which is consistent to the spread characteristics in the model in Easley and O'Hara (1987) and Glosten and Milgrom (1985).

Table 3. Regression Analysis: The characteristics of the component covariances of the transactional return covariance, and $Roll\_Spread$

dependent \ independent	Intercept	$MktCap_i$	$VarSpread_i$	$AsyFreqRatio_i$	Adj. R^2	No. obs
$Roll\_Spread_i \geq 0$	0.0023* (160.81)	-1.89E-11* (-44.26)	21.5921* (6.63)	-0.0006* (-20.38)	0.2663	545,443
$Cov(r^M_{i,t}, r^M_{i,t+1})$	-1.02E-6* (-14.97)	-1.03E-17 (-0.32)	-0.1086* (-4.09)	1.43E-6* (5.19)	0.2215	558,391
$Cov(r^M_{i,t}, \delta^M_{i,t+1})$	1.03E-6* (14.05)	1.39E-15 (0.49)	0.0907* (3.79)	-1.44E-6* (-5.77)	0.1498	558,391
$Cov(\delta^M_{i,t}, r^M_{i,t+1})$	2.15E-6* (22.89)	-9.32E-15* (-2.59)	0.1686* (5.63)	-1.87E-6* (-6.13)	0.3121	558,391
$Cov(\delta^M_{i,t}, \delta^M_{i,t+1})$	-3.51E-6* (-32.85)	1.53E-14* (4.54)	-0.1724* (-6.10)	2.96E-6* (10.29)	0.2759	558,391

* Significant at 1% level or higher.

We regress daily-firm $Cov(r^M_{i,t}, r^M_{i,t+1})$, $Cov(r^M_{i,t}, \delta^M_{i,t+1})$, $Cov(\delta^M_{i,t}, r^M_{i,t+1})$, $Cov(\delta^M_{i,t}, \delta^M_{i,t+1})$, and *Roll* spread, respectively, on the daily mean market capitalization, $MktCap_i$, the daily frequency ratio of the trade-direction continuation over discontinuation at the natural logarithm level, $AsyFreqRatio_i$ and the variance of the effective percentage spread, $VarSpread_i$. The computation of the effective spread is based upon the method in Peterson and Fialkowski (1994). The form of the regressions is as the following:

$$Dependent_i = \alpha_0 + \alpha_1 \cdot MktCap_i + \alpha_2 \cdot VarSpread_i + \alpha_3 \cdot AsyFreqRatio_i + \varepsilon_i .$$

Heteroscedasticity and autocorrelation consistent t-values (Newey and West, 1987) are reported in the parentheses. All the variables used in the regressions are all at the daily frequency level by firm. We employ the NYSE TAQ data and only select firms listed on the NYSE and only consider firms with daily mean prices greater than $1 and less than $500. The sample period is from January to December 2003.

3. Portfolio Covariance and Its Covariance Components

Differential serial correlation patterns between the individual return and portfolio return are well-known (Lo and MacKinlay, 1990). In this section, we perform empirical estimation to verify the consistency of the portfolio return behavior in term of our model predictions. We form portfolios with various trading lengths. The motive of the portfolio analysis here is inspired by French and Roll (1986) that the return volatility is caused by public and private information. Private information incorporates into prices through trading activities. The arrival of public information itself affects security prices without trading activities, but trading activities themselves confirm the significance of public information. Hence, the trading activity can be a crucial indicator that portraits the scope, intensity, and direction of the flow of information (Jones, Kaul, and Lipson, 1994). Given the purpose aforementioned, we need to form the portfolios that are able to seize those information-carrying stocks whose information impacts not only themselves but also each other. Furthermore, due to private information is perishable (e.g., Admati and Pfleiderer, 1988; Easley and O'Hara, 1987; Kyle, 1985), we focus on the flow of information within a day. We need to form the daily intraday portfolios that enable us to pursue our goals. To form these daily intraday portfolios, there are two seemingly contradicted sample selection criteria: The shorter the intraday interval, e.g., the one-min trading interval, the more observations per day we are able to obtain from a qualified stock in our daily sample. However, this also means that a smaller number of the qualified stock will be included in our sample. More observations obtained from a single stock per day enable us to examine trading activities in a greater detail. However, given this is a portfolio analysis, we also need to expand the sample scope in order to catch the essence of the cross-autocorrelation among stocks in Lo and MacKinlay (1990). Hence, our effort on the sample selection is to obtain as many individual security's daily intraday observations and the total number of securities in our sample as possible. There are a total of six and half hours (390 minutes) in the regular trading period on a typical trading day on the New York Stock Exchange. If we use one-min trading interval as the sample selection criterion, then to satisfy this criterion a stock needs to trade at least once per minute and to have a total of

391 price observations selected on a trading day[10]. Nevertheless, few securities are able to have this kind of high and even trading frequency in a given trading day. Thus, the one-min trading interval portfolio will only contain a handful of securities on a typical trading day and they are less likely to offer a good daily estimation of the cross-correlations that portrait the flow of information in the market. Toward the other end of the spectrum, if we choose to use 30-min trading interval as the sample selection criterion, then per day a stock trades at least once every 30 minutes will need to have 14 price observations selected. It means that 13 30-min returns can be calculated and only have a degree of freedom of 12 in estimating the return serial correlation. In all considerations, thus, we decide to form six equally-weighted portfolios with various lengths of trading interval, i.e., 5-min, 10-min, 13-min, 15-min, 26-min and 30-min trading intervals, under daily basis. The 5-min trading interval portfolios have a highest daily number of observations per stock, i.e., 79 5-min price observations, but the lowest number of constituent stocks. In contrast, the 30-min trading interval portfolios have a lowest daily number of observations per stock, i.e., 14 30-min price observations, but a highest number of constituent stocks. In each type of the portfolios, its constituent stocks all have a common feature, i.e., they all traded at least every n-min on a trading day. This common feature is the key to establish the test environment needed to investigate the time series behaviors of the liquidity return, r_t^q and the mid-point return, r_t^M, within individual securities and among each other.

[10] For six and half hours trading period, there are 390 one-min trading intervals. To have 390 one-min returns, we will need to have 391 observations. We will select the first trade price in each one-min interval and then select the last trade price in the last trading interval. We make sure the selected first trade price and the last trade price in the last trading interval are not from the same trade. Thus, we have 391 trade prices over 390 one-min trading intervals.

Table 4. Portfolio Return Autocorrelation Decomposition I

covariances / trading intervals	$Cov(R_t, R_{t+1})$ $Corr(R_t, R_{t+1})$	$Cov(R_t^M, R_{t+1}^M)$ $Corr(R_t^M, R_{t+1}^M)$	$Cov(R_t^{q^M}, R_{t+1}^M)$ $Corr(R_t^{q^M}, R_{t+1}^M)$	$Cov(R_t^M, R_{t+1}^{q^M})$ $Corr(R_t^M, R_{t+1}^{q^M})$	$Cov(R_t^{q^M}, R_{t+1}^{q^M})$ $Corr(R_t^{q^M}, R_{t+1}^{q^M})$
5-min trading interval:	5.5948E-8 (6.3592E-9)	5.4564E-8 (6.0185E-9)	2.8500E-8 (1.6748E-9)	-1.9961E-8 (1.2675E-9)	-7.1556E-9 (4.5317E-10)
124,800 daily-firms	0.0997 (0.0080)	0.0995 (0.0079)	0.2824 (0.0084)	-0.2055 (0.0081)	-0.4241 (0.0066)
10-min trading interval:	8.5947E-8 (1.5670E-8)	8.1133E-8 (1.5989E-8)	3.3491E-8 (2.5048E-9)	-2.2543E-8 (1.8366E-9)	-6.135E-9 (4.4342E-10)
259,754 daily-firms	0.0835 (0.0123)	0.0836 (0.0125)	0.2644 (0.0129)	-0.2180 (0.0109)	-0.3779 (0.0109)
13-min trading interval:	9.8075E-8 (2.3076E-8)	9.416E-8 (2.2524E-8)	3.451E-8 (2.8938E-9)	-2.4343E-8 (2.2397E-9)	-6.2527E-9 (4.0222E-10)
304,294 daily-firms	0.0648 (0.0142)	0.0648 (0.0142)	0.2348 (0.0149)	-0.1903 (0.0128)	-0.3933 (0.0121)
15-min trading interval:	11.2000E-8 (2.7985E-8)	9.8109E-8 (2.7788E-8)	4.0376E-8 (3.3708E-9)	-2.0286E-8 (2.2237E-9)	-6.2195E-9 (3.6457E-10)
325,717 daily-firms	0.0711 (0.0145)	0.0657 (0.0147)	0.2519 (0.0158)	-0.1532 (0.0137)	-0.3804 (0.0124)
26-min trading interval:	9.6885E-8 (5.4515E-8)	10.2000E-8 (5.3702E-8)	4.383E-8 (5.4676E-9)	-4.2615E-8 (4.423E-9)	-6.4485E-9 (5.5234E-10)
391,944 daily-firms	0.0175 (0.0184)	0.0207 (0.0185)	0.1780 (0.0208)	-0.2107 (0.0174)	-0.3224 (0.0167)
30-min trading interval:	-4.9248E-8 (6.5633E-8)	-3.7546E-8 (6.407E-8)	6.2597E-8 (6.8148E-9)	-6.6495E-8 (7.2946E-9)	-7.8046E-9 (6.8526E-10)
407,494 daily-firms	0.0003 (0.0191)	0.0027 (0.0191)	0.2454 (0.0220)	-0.2649 (0.0183)	-0.2963 (0.0182)

We decompose the equally-weighted portfolio return covariance, $Cov(R_t, R_{t+1})$, into four parts; the portfolio mid-point return covariance, $Cov(R_t^M, R_{t+1}^M)$, the covariance of portfolio liquidity return, $Cov(R_t^{q^M}, R_{t+1}^{q^M})$, the covariance between the lead portfolio mid-point return and the lagged portfolio liquidity return, $Cov(R_t^{q^M}, R_{t+1}^M)$, and the covariance between the lead portfolio liquidity return and the lagged portfolio mid-point return, $Cov(R_t^M, R_{t+1}^{q^M})$. In each cell, there are four statistics, the first two statistics are the mean daily portfolio return covariance and its standard error of the mean, and the remaining two statistics are the mean daily portfolio return correlation and its standard error of the mean. Our equally-weighted market portfolios include the transactional data of

NYSE stocks that are retrieved from January 2003 to December 2003, NYSE TAQ database. We form six portfolios based upon the length of the trading interval, i.e., 5-, 10-, 13-, 15-, 26-, and 30-min trading intervals. Under the daily basis, we only select stocks that have full trading interval return observations. For instance, there are a total of 78 return observations in the 5-min intrady trading interval per day. We first estimate all relevant statistics and then average them across the entire sample period.

Table 4 presents the estimates of the six equally-weighted portfolio covariances, $Cov(R_t, R_{t+1})$, that have various trading interval returns and the four decomposed portfolio covariance components, i.e., $Cov(R_t^M, R_{t+1}^M)$, $Cov(R_t^{q^M}, R_{t+1}^M)$, $Cov(R_t^M, R_{t+1}^{q^M})$, and $Cov(R_t^{q^M}, R_{t+1}^{q^M})$, respectively. These four empirical portfolio covariance components are to proxy $Cov(R_t^*, R_{t+1}^*)$, $Cov(R_t^q, R_{t+1}^*)$, $Cov(R_t^*, R_{t+1}^q)$, and $Cov(R_t^q, R_{t+1}^q)$, respectively, described in Eq.(10). In addition, to help our analysis described in section 2.2, each of these four portfolio covariance components can be further decomposed into the portfolio self-covariance, i.e., the elements of security i covary within the security, and the portfolio cross-covariance, i.e., the elements of security i covary with the elements of other securities. Table 5 presents these portfolio self-covariances and portfolio cross-covariance. In Table 4, the mean of the portfolio mid-point return serial covariance, $Cov(R_t^M, R_{t+1}^M)$, of the 5-min trading interval is positive. However, according to Eq.(11), $Cov(R_t^M, R_{t+1}^M)$ contains two different forces and can be decomposed into the self- and cross-covariances. As expected in Table 5, the former is negative by reflecting the bid-ask bounce effect and the latter enjoys a positive sign by having the "commonality in private information" of the *second kind* that is a cross spillover of private information from the spread (at time t) of security i to those of other securities at time $t+1$. The magnitude of this cross-covariance is much larger than that of the self-covariance. Thus, in Table 4 we observe a positive $Cov(R_t^M, R_{t+1}^M)$ and this result is consistent in all six portfolios with various lengths of trading interval.

Table 5. Portfolio Return Autocorrelation Decomposition II

Intraday subgroups	$\sum_{i=1}^{n} Cov(r^M_{i,t}, r^M_{i,t+1})$	$\sum_{i=1}^{n}\sum_{j=1, j\neq i}^{n} Cov(r^M_{i,t}, r^M_{j,t+1})$	$\sum_{i=1}^{n} Cov(\delta^M_{i,t}, \delta^M_{i,t+1})$	$\sum_{i=1}^{n}\sum_{j=1, j\neq i}^{n} Cov(\delta^M_{i,t}, \delta^M_{j,t+1})$
Panel A:				
5-min trading interval: 124,800 daily-firms	-2.7079E-10 (4.7439E-11)	5.4835E-8 (6.0058E-9)	-9.5079E-10 (5.0885E-11)	-6.2048E-9 (4.3031E-10)
10-min trading interval: 259,754 daily-firms	-4.5492E-10 (2.9883E-11)	8.1588E-8 (1.5974E-8)	-7.2097E-10 (2.2749E-11)	-5.4141E-9 (4.3227E-10)
13-min trading interval: 304,294 daily-firms	-6.2733E-10 (3.5886E-11)	9.4788E-8 (2.2505E-8)	-7.7695E-10 (2.4629E-11)	-5.4757E-9 (3.9274E-10)
15-min trading interval: 325,717 daily-firms	-7.2211E-10 (3.8233E-11)	9.8831E-8 (2.7764E-8)	-8.3517E-10 (2.2514E-11)	-5.3843E-9 (3.5315E-10)
26-min trading interval: 391,944 daily-firms	-1.2975E-9 (6.6071E-11)	10.3000E-8 (5.3655E-8)	-1.1621E-9 (3.5345E-11)	-5.2863E-9 (5.4001E-10)
30-min trading interval: 407,494 daily-firms	-1.7594E-9 (8.3832E-11)	-3.5786E-8 (6.4006E-8)	-1.2874E-9 (3.7503E-11)	-6.5172E-9 (6.7120E-10)
Panel B:				
Intraday subgroups	$\sum_{i=1}^{n} Cov(\delta^M_{i,t}, r^M_{i,t+1})$	$\sum_{i=1}^{n}\sum_{j=1, j\neq i}^{n} Cov(\delta^M_{j,t}, r^M_{i,t+1})$	$\sum_{i=1}^{n} Cov(r^M_{i,t}, \delta^M_{i,t+1})$	$\sum_{i=1}^{n}\sum_{j=1, j\neq i}^{n} Cov(r^M_{j,t}, \delta^M_{i,t+1})$
5-min trading interval: 124,800 daily-firms	7.2089E-10 (4.7901E-11)	2.7779E-8 (1.6575E-9)	1.5324E-10 (4.0811E-11)	-2.0114E-8 (1.2697E-9)
10-min trading interval: 259,754 daily-firms	5.2813E-10 (1.9358E-11)	3.2963E-8 (2.4975E-9)	1.6557E-10 (1.4133E-11)	-2.2708E-8 (1.8344E-9)
13-min trading interval: 304,294 daily-firms	5.6280E-10 (2.1627E-11)	3.3947E-8 (2.8872E-9)	2.1301E-10 (1.7692E-11)	-2.4556E-8 (2.2388E-9)
15-min trading interval: 325,717 daily-firms	5.8559E-10 (1.8315E-11)	3.9791E-8 (3.3637E-9)	2.4195E-10 (1.4030E-11)	-2.0528E-8 (2.2238E-9)
26-min trading interval: 391,944 daily-firms	7.8976E-10 (2.8137E-11)	4.3041E-8 (5.458E-9)	3.3802E-10 (2.2879E-11)	-4.2953E-8 (4.4223E-9)
30-min trading interval: 407,494 daily-firms	8.7979E-10 (3.2505E-11)	6.1718E-8 (6.8029E-9)	3.7767E-10 (2.3928E-11)	-6.6873E-8 (7.2901E-9)

By assuming the bid-ask spread symmetry, we further decompose portfolio $Cov(R_t^M, R_{t+1}^M)$ into $\sum_{i=1}^{n} Cov(r_{i,t}^M, r_{i,t+1}^M)$ and $\sum_{i=1}^{n}\sum_{j=1, j\neq i}^{n} Cov(r_{i,t}^M, r_{j,t+1}^M)$; portfolio $Cov(R_t^{q^M}, R_{t+1}^{q^M})$ into $\sum_{i=1}^{n} Cov(\delta_{i,t}^M, \delta_{i,t+1}^M)$ and $\sum_{i=1}^{n}\sum_{j=1, j\neq i}^{n} Cov(\delta_{i,t}^M, \delta_{j,t+1}^M)$; portfolio $Cov(R_t^{q^M}, R_{t+1}^M)$ into $\sum_{i=1}^{n} Cov(\delta_{i,t}^M, r_{i,t+1}^M)$ and $\sum_{i=1}^{n}\sum_{j=1, j\neq i}^{n} Cov(\delta_{i,t}^M, r_{j,t+1}^M)$; portfolio $Cov(R_t^M, R_{t+1}^{q^M})$ into $\sum_{i=1}^{n} Cov(r_{i,t}^M, \delta_{i,t+1}^M)$ and $\sum_{i=1}^{n}\sum_{j=1, j\neq i}^{n} Cov(r_{i,t}^M, \delta_{j,t+1}^M)$. Our equally-weighted market portfolios include the transactional data of NYSE stocks that are retrieved from January to December 2003, NYSE TAQ database. We form five equally-weighted portfolios based upon the length of the trading interval, i.e., 5-, 10-, 13-, 15-, 26-, and 30-min trading intervals. Under the daily basis, we only select stocks that have full trading interval return observations. For instance, there are 78 return observations in the 5-min intraday trading interval per day. We first estimate all relevant statistics and then average them across the entire sample period. All covariances and their standard errors (in the parentheses).

The portfolio covariance of the lead liquidity return and lagged true return, $Cov(R_t^M, R_{t+1}^{q^M})$, is negative in Table 4. As shown in Eq.(12), it also contains a self-covariance, described in (12a) and a cross-covariance, described in (12b). This self-covariance, $\sum_{i=1}^{n} -[Cov(s_{i,t-1}^{AS} \cdot Q_{i,t-1}, s_{i,t}^{AS} \cdot Q_{i,t}) + Cov(s_{i,t-1}^{AS} \cdot Q_{i,t-1}, s_{i,t}^{GP} \cdot Q_{i,t})]$, has two components all involving the bid-ask bounce effect. Since having a negative sign in front of them, this set of self-covariance shows positive in Table 5. The cross-covariance, $\sum_{i=1}^{n}\sum_{j=1, j\neq i}^{n} -Cov(s_{i,t-1}^{AS} \cdot Q_{i,t-1}, s_{j,t}^{AS} \cdot Q_{j,t})$, is negative because it has a negative sign in front of it, even though it holds the feature of the "commonality in private information" of the *second kind* that should be positive in value. Nonetheless, due to the magnitude of this cross-covariance is larger than that of the self-covariances, it leads $Cov(R_t^M, R_{t+1}^{q^M})$ to be negative, which implies that in $Cov(R_t^M, R_{t+1}^{q^M})$ the influence of the "commonality in private information" of the *second kind* is greater than that of the bid-ask bounce effect. This is the same phenomenon that we have observed in the mean of the portfolio mid-point return serial covariance, $Cov(R_t^M, R_{t+1}^M)$.

In Table 4, the mean of the portfolio covariance of the lead true return and lagged liquidity return, $Cov(R_t^{q^M}, R_{t+1}^M)$, is positive. It also can be further decomposed into two sets of covariance components, i.e., the self-covariance and the cross-covariance described in Eq.(13). The self-covariance, $\sum_{i=1}^{n} Cov(\delta_{i,t}^M, r_{i,t+1}^M)$, is positive, as predicted, because it contains a variance of $s_{i,t}^{AS} \cdot Q_{i,t}$ and the bid-ask bounce effect, $\sum_{i=1}^{n} -[Cov(s_{i,t-1}^{AS} \cdot Q_{i,t-1}, s_{i,t}^{AS} \cdot Q_{i,t}) + Cov(s_{i,t-1}^{GP} \cdot Q_{i,t-1}, s_{i,t}^{AS} \cdot Q_{i,t})]$, which has a negative sign in front of it. $Var(s_{i,t}^{AS} \cdot Q_{i,t})$ contained in the self-covariance $\sum_{i=1}^{n} Cov(\delta_{i,t}^M, r_{i,t+1}^M)$ implies there is a private information transfer from the liquidity return, $\delta_{i,t}$, to the true return, $r_{i,t+1}^M$ within each security i across time to make $\sum_{i=1}^{n} Cov(\delta_{i,t}^M, r_{i,t+1}^M)$ positive, and it is camouflaged by the bid-ask bounce effect. This is the same phenomenon found in Table 2 for Eq.(8).

The cross-covariance $\sum_{i=1}^{n} \sum_{j=1, j\neq i}^{n} Cov(\delta_{i,t}^M, r_{j,t+1}^M)$ is positive (Table 4), which implies that the influence of the "commonality in private information" of the *first kind* (contemporaneous influence) is stronger than that of the "commonality in private information" of the *second kind* (cross-security lead-lag influence). The "commonality in private information" of the *first kind* reflects the common insight of market-wide influence and the "commonality in private information" of the *second kind* reflects the special insight of market-wide influence. These findings imply that on average common insight dominates the unique insight. Perhaps, to have a special ability is relatively rare comparing to have a common ability.

Finally, in Table 4, the portfolio serial liquidity return covariance, $Cov(R_t^{q^M}, R_{t+1}^{q^M})$, is negative in all six portfolios of various interval lengths. It also contains two sets of covariance components, i.e., the self-covariance and the cross-covariance (see Eq.(14) and Table 5). From Eq.(14), the self-covariance $\sum_{i=1}^{n} Cov(\delta_{i,t}^M, \delta_{i,t+1}^M)$ can be further decomposed into three covariance components: $\sum_{i=1}^{n} Cov(s_{i,t} \cdot Q_{i,t}, s_{i,t+1} \cdot Q_{i,t+1})$ and $\sum_{i=1}^{n} Cov(s_{i,t-1} \cdot Q_{i,t-1}, s_{i,t} \cdot Q_{i,t})$ involve the bid-ask bounce effect and $-\sum_{i=1}^{n} Var(s_{i,t} \cdot Q_{i,t})$ is the variance of the one-sided spread, with a negative sign in front of it. Here, $-\sum_{i=1}^{n} Var(s_{i,t} \cdot Q_{i,t})$ is a measure of trading noise, the same measure described in Eq.(9) for individual securities. The cross-covariance $\sum_{i=1}^{n} \sum_{j=1, j \neq i}^{n} Cov(\delta_{i,t}, \delta_{j,t+1})$ is also negative (Table 5) and comprised of three covariance components: $\sum_{i=1}^{n} \sum_{j=1, j \neq i}^{n} Cov(s_{i,t}^{AS} \cdot Q_{i,t}, s_{j,t+1}^{AS} \cdot Q_{j,t+1})$, $\sum_{i=1}^{n} \sum_{j=1, j \neq i}^{n} Cov(s_{i,t-1}^{AS} \cdot Q_{i,t-1}, s_{j,t}^{AS} \cdot Q_{j,t})$, and $\sum_{i=1}^{n} \sum_{j=1, j \neq i}^{n} - Cov(s_{i,t}^{AS} \cdot Q_{i,t}, s_{j,t}^{AS} \cdot Q_{j,t})$. The first two covariance components involve the feature of the "commonality in private information" of the *second kind* and the third component covariance has the feature of the "commonality in private information" of the *first kind* (with a negative sign in front of it). If on average common insight (the *first kind*) is more influential than the special insight (the *second kind*), then we should observe a same negative sign on the cross-covariance $\sum_{i=1}^{n} \sum_{j=1, j \neq i}^{n} Cov(\delta_{i,t}^M, \delta_{j,t+1}^M)$ in Eq.(14). In Table 5, the mean of the cross-covariance $\sum_{i=1}^{n} \sum_{j=1, j \neq i}^{n} Cov(\delta_{i,t}^M, \delta_{j,t+1}^M)$ in Eq.(14) is negative in all six portfolios of various interval lengths.

V. CONCLUSION

This study investigates the security value updating process with respect to private information. We first decompose the transaction return of individual securities into the true return and the liquidity return. We then construct a covariance scheme to show that the individual security value updating process runs from the lagged liquidity return to the lead true return and the key element is the volatility of the one-sided adverse-selection component, which is camouflaged by trading noise. The portfolio value updating process mainly involves both the contemporaneous and lead-lag cross-security covariances of the adverse-selection components among the constituent securities. These covariances are termed the "commonality in private information" of the *first kind* and the *second kind*, respectively. Furthermore, this study shows that the influence of the *first kind* eclipses that of the *second kind*. Among these trails of private information transmission, we also find that in the spread the gross-profit component is not the only source of trading noise, the adverse-selection component can also amplify trading noise, when it is across time and across securities. These findings are helpful in addressing the question posted by Lo and MacKinlay (1990) regarding the economic sources of positive cross-autocorrelation across securities.

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APPENDIX A: APPLICATION OF DELTA METHOD

Based upon the Delta method on Chapter 5 (pp. 243-244), George Casella & Roger Berger, Statistical Inference, 2001 Second Edition, Duxbury Press

Let $s_{i,t-1}^{AS} \cdot Q_{i,t-1} + \varepsilon_{i,t}$ be x and $f(x) = \ln(1+x) \in C(\infty)$, we express its Taylor expansion at zero as follow:

$$\begin{aligned} f(x) &= f(0) + \frac{f'(0)}{1!}(x-0) + \frac{f''(0)}{2!}(x-0)^2 + \frac{f'''(0)}{3!}(x-0)^3 + \ldots \\ &= \ln(1) + \frac{1}{1!(1+0)}x - \frac{1}{2!(1+0)^2}x^2 + \frac{f'''(0)}{3!(1+0)^3}x^3 + \ldots \\ &= 0 + x - \frac{x^2}{2!} + \frac{2x^3}{3!} + \ldots \\ &= x + \text{Remainder} \end{aligned} \tag{A1}$$

When x is small, as $s_{i,t-1}^{AS}$ and $\varepsilon_{i,t}$ are percentages of the true price $p_{i,t-1}$ for security i a time $t-1$, we can further degenerate (A1) into $f(x) \approx x$.

Thus,

$$\begin{aligned} f(s_{i,t-1}^{AS} \cdot Q_{i,t-1} + \varepsilon_{i,t}) &= \ln(1 + s_{i,t-1}^{AS} \cdot Q_{i,t-1} + \varepsilon_{i,t}) \\ &\approx s_{i,t-1}^{AS} \cdot Q_{i,t-1} + \varepsilon_{i,t} \end{aligned}$$

Q.E.D.

APPENDIX B: COVARIANCE OF $s_{i,t}^{GP}$ AND $s_{i,t}^{AS}$

$$
\begin{aligned}
& Cov(s_{i,t}^{GP} \cdot Q_{i,t}, s_{i,t}^{AS} \cdot Q_{i,t}) \\
&= E[s_{i,t}^{GP} \cdot s_{i,t}^{AS} \cdot Q_{i,t}^2] - E[s_{i,t}^{GP} \cdot Q_{i,t}] \cdot E[s_{i,t}^{AS} \cdot Q_{i,t}] \\
&= E[s_{i,t}^{GP} \cdot s_{i,t}^{AS}] - E[s_{i,t}^{GP}]E[Q_{i,t}] \cdot E[s_{i,t}^{AS}] \cdot E[Q_{i,t}], \qquad \text{if } Q_{i,t} \perp s_{i,t}^{AS}, Q_{i,t} \perp s_{i,t}^{GP} \\
&= E[s_{i,t}^{GP}] \cdot E[s_{i,t}^{AS}] \cdot (1 - E[Q_{i,t}]^2), \qquad \text{if } s_{i,t}^{AS} \perp s_{i,t}^{GP},
\end{aligned}
\tag{B1}
$$

where $s_{i,t}^{AS}$ is the adverse-selection component, $s_{i,t}^{GP}$ is the gross-profit component in the bid-ask spread, and $Q_{i,t}$ is the trade direction indicator, $Q_{i,t} = +1$, if it is a buyer-initiated trade, and $Q_{i,t} = -1$, if a seller-initiated trade.

From Eq.(B1), we know that $Cov(s_{i,t}^{GP} \cdot Q_{i,t}, s_{i,t}^{AS} \cdot Q_{i,t}) \geq 0$, because $0 \leq E[Q_{i,t}]^2 \leq +1$.

$E[Q_{i,t}] = 0$, if the probabilities of $Q_{i,t} = +1$ and $Q_{i,t} = -1$ are symmetric. Otherwise,

$-1 \leq E[Q_{i,t}] \leq +1$, which leads to $0 < E[Q_{i,t}]^2 \leq +1$. **Q.E.D.**

APPENDIX C: ROLL'S COVARIANCE IN PORTFOLIO APPLICATION

In this section, we wish to examine whether there is a "cross-firm" lead-lag bid-ask bounce effect at the portfolio level that is similar to that of individual security in Roll (1984).

Based on the assumptions of Roll (1984): (i) There is no new information in the market, i.e., ask and bid prices are constant and the bid-ask spread s is also constant, (ii) the market is semi-strong form efficient, (iii) the price changes of all firms are stationary. We relax the probability of transaction price occurrence by assigning α is the probability of transacting at ask price for security i at time t (from $t-1$) and β is the probability of transacting at ask price for security j at time $t+1$ (from $t-1$), where $\alpha \perp \beta$. As in Roll (1984), in order to obtain the serial covariance, we need three time points, i.e., $t-1$, t, and $t+1$, to come up two price changes, i.e., $\Delta P_{i,t} \in \{-s_i, 0, +s_i\}$ and $\Delta P_{j,t+1} \in \{-s_j, 0, +s_j\}$. Due to the initial location of the transaction prices of security i (at ask or bid at time $t-1$) and j (at ask or bid at time t) need to be identified first before any subsequent price change can be determined, we have a total of four sets of probability scenarios to describe them and then we combine them into a single probability table that facilitates us to derive the needed statistics for cross-firm price changes. For security i, transacting at ask the probability is α, and at bid the probability is $1-\alpha$. For security j, transacting at ask the probability is β, and at bid the probability is $1-\beta$.

1. $P_{i,t-1} = Ask$ (probability = α) and $P_{j,t} = Ask$ (probability = β): the joint probability = $\alpha\beta$

	$\Delta P_{i,t} = -s_i$	$\Delta P_{i,t} = 0$	$\Delta P_{i,t} = +s_i$
$\Delta P_{j,t+1} = -s_j$	(1-α)(1-β) = $(1-\alpha)_{i,t,\ bid(-s)} \times (1-\beta)_{j,t+1,bid(-s)}$	α(1-β) = $\alpha_{i,t,\ bid(-s)} \times (1-\beta)_{j,t+1,bid(-s)}$	**0.0** = $0.0_{i,t,\ bid(+s)} \times (1-\beta)_{j,t+1,bid(-s)}$
$\Delta P_{j,t+1} = 0$	(1-α) β = $(1-\alpha)_{i,t,\ bid(-s)} \times \beta_{j,t+1,bid(0)}$	Aβ = $\alpha_{i,t,\ bid(-s)} \times \beta_{j,t+1,bid(0)}$	**0.0** = $0.0_{i,t,\ bid(+s)} \times \beta_{j,t+1,bid(0)}$
$\Delta P_{j,t+1} = +s_j$	**0.00** = $(1-\alpha)_{i,t,bid(-s)} \times 0.0_{j,t+1,ask(+s)}$	**0.00** = $\alpha_{i,t,\ bid(-s)} \times 0.0_{j,t+1,ask(+s)}$	**0.0** = $0.0_{i,t,\ bid(+s)} \times 0.0_{j,t+1,ask(+s)}$

2. $P_{i,t-1} = Ask$ (probability = α) and $P_{j,t} = Bid$ (probability = $1-\beta$): the joint probability = $\alpha(1-\beta)$

	$\Delta P_{i,t} = -s_i$	$\Delta P_{i,t} = 0$	$\Delta P_{i,t} = +s_i$
$\Delta P_{j,t+1} = -s_j$	0.00 = $(1-\alpha)_{i,t,\ bid(-s)} \times 0.0_{j,\ t+1,bid(-s)}$	0.00 = $\alpha_{i,t,\ bid(-s)} \times 0.0_{j,\ t+1,bid(-s)}$	**0.0** = $0.0_{i,t,\ bid(+s)} \times 0.0_{j,\ t+1,bid(-s)}$
$\Delta P_{j,t+1} = 0$	(1-α) (1-β) = $(1-\alpha)_{i,t,\ bid(-s)} \times (1-\beta)_{j,t+1,bid(0)}$	α(1-β) = $\alpha_{i,t,\ bid(-s)} \times (1-\beta)_{j,t+1,bid(0)}$	**0.0** = $0.0_{i,t,bid(+s)} \times (1-\beta)_{j,t+1,bid(0)}$
$\Delta P_{j,t+1} = +s_j$	(1-α) β = $(1-\alpha)_{i,t,bid(-s)} \times \beta_{j,t+1,ask(+s)}$	Aβ = $\alpha_{i,t,bid(-s)} \times \beta_{j,t+1,ask(+s)}$	**0.0** = $0.0_{i,t,bid(+s)} \times \beta_{j,t+1,ask(+s)}$

3. $P_{i,t-1} = Bid$ (probability = $1-\alpha$) and $P_{j,t} = Ask$ (probability = β): the joint probability = $(1-\alpha)\beta$

	$\Delta P_{i,t} = -s_i$	$\Delta P_{i,t} = 0$	$\Delta P_{i,t} = +s_i$
$\Delta P_{j,t+1} = -s_j$	**0.00** = $0.0_{i,t,\ bid(-s)} \times (1-\beta)_{j,\ t+1,bid(-s)}$	(1-α)(1-β) = $(1-\alpha)_{i,t,\ bid(-s)} \times (1-\beta)_{j,\ t+1,bid(-s)}$	α(1-β) = $\alpha_{i,t,\ bid(+s)} \times (1-\beta)_{j,\ t+1,bid(-s)}$
$\Delta P_{j,t+1} = 0$	**0.00** = $0.0_{i,t,\ bid(-s)} \times \beta_{j,t+1,bid(0)}$	(1-α)β = $(1-\alpha)_{i,t,\ bid(-s)} \times \beta_{j,t+1,bid(0)}$	αβ = $\alpha_{i,t,\ bid(+s)} \times \beta_{j,t+1,bid(0)}$
$\Delta P_{j,t+1} = +s_j$	**0.00** = $0.0_{i,t,bid(-s)} \times 0.0_{j,t+1,ask(+s)}$	**0.00** = $(1-\alpha)_{i,t,\ bid(-s)} \times 0.0_{j,t+1,ask(+s)}$	**0.00** = $\alpha_{i,t,\ bid(+s)} \times 0.0_{j,t+1,ask(+s)}$

4. $P_{i,t-1} = Bid$ (probability = $1-\alpha$) and $P_{j,t} = Bid$ (probability = $1-\beta$): the joint probability = $(1-\alpha)(1-\beta)$

	$\Delta P_{i,t} = -s_i$	$\Delta P_{i,t} = 0$	$\Delta P_{i,t} = +s_i$
$\Delta P_{j,t+1} = -s_j$	**0.00** = $0.0_{i,t,\ bid(-s)} \times 0.0_{j,\ t+1,bid(-s)}$	**0.00** = $(1-\alpha)_{i,t,\ bid(-s)} \times 0.0_{j,\ t+1,bid(-s)}$	**0.00** = $\alpha_{i,t,\ bid(+s)} \times 0.0_{j,\ t+1,bid(-s)}$
$\Delta P_{j,t+1} = 0$	**0.00** = $0.0_{i,t,\ bid(-s)} \times (1-\beta)_{j,t+1,bid(0)}$	(1-α)(1-β) = $(1-\alpha)_{i,t,\ bid(-s)} \times (1-\beta)_{j,t+1,bid(0)}$	α(1-β) =$\alpha_{i,t,\ bid(+s)} \times (1-\beta)_{j,t+1,bid(0)}$
$\Delta P_{j,t+1} = +s_j$	**0.00** = $0.0_{i,t,bid(-s)} \times \beta_{j,t+1,ask(+s)}$	(1-α)β = $(1-\alpha)_{i,t,\ bid(-s)} \times \beta_{j,t+1,ask(+s)}$	αβ =$\alpha_{i,t,bid(+s)} \times \beta_{j,t+1,ask(+s)}$

Finally, we present the combined effect of (1), (2), (3), and (4):

	$\Delta P_{i,t} = -s_i$	$\Delta P_{i,t} = 0$	$\Delta P_{i,t} = +s_i$	Marginal probability
$\Delta P_{j,t+1} = -s_j$	$P_{j=-s}^{i=-s}$ $= (1-\alpha)(1-\beta)(\alpha\beta)$	$P_{j=-s}^{i=0} =$ $\alpha^2\beta(1-\beta)$ $+ (1-\alpha)^2\beta(1-\beta)$	$P_{j=-s}^{i=+s} =$ $(1-\alpha)(1-\beta)\alpha\beta$	$MP_{j,t+1}^{-s} =$ $2(1-\alpha)(1-\beta)(\alpha\beta)$ $+ \alpha^2\beta(1-\beta)$ $+ (1-\alpha)^2\beta(1-\beta)$
$\Delta P_{j,t+1} = 0$	$P_{j=0}^{i=-s} =$ $(1-\alpha)\alpha\beta^2 +$ $\alpha(1-\alpha)(1-\beta)^2$	$P_{j=0}^{i=0} =$ $(\alpha\beta)^2 + [\alpha(1-\beta)]^2$ $+ [(1-\alpha)\beta]^2$ $+ [(1-\alpha)(1-\beta)]^2$	$P_{j=0}^{i=+s} =$ $(1-\alpha)\alpha\beta^2$ $+ \alpha(1-\alpha)(1-\beta)^2$	$MP_{j,t+1}^{0} =$ $(1-\alpha)\alpha\beta^2 +$ $\alpha(1-\alpha)(1-\beta)^2$ $+ (\alpha\beta)^2 + [\alpha(1-\beta)]^2$ $+ [(1-\alpha)\beta]^2$ $+ [(1-\alpha)(1-\beta)]^2$ $+ (1-\alpha)\alpha\beta^2$ $+ \alpha(1-\alpha)(1-\beta)^2$
$\Delta P_{j,t+1} = +s_j$	$P_{j=+s}^{i=-s} =$ $(1-\alpha)(1-\beta)\alpha\beta$	$P_{j=+s}^{i=0} =$ $\alpha^2\beta(1-\beta)$ $+ (1-\alpha)^2\beta(1-\beta)$	$P_{j=+s}^{i=+s} =$ $(1-\alpha)(1-\beta)\alpha\beta$	$MP_{j,t+1}^{+s} =$ $2(1-\alpha)(1-\beta)\alpha\beta$ $+ \alpha^2\beta(1-\beta)$ $+ (1-\alpha)^2\beta(1-\beta)$
Marginal probability	$MP_{i,t}^{-s} =$ $(1-\alpha)(1-\beta)\alpha\beta$ $+ (1-\alpha)\alpha\beta^2 +$ $\alpha(1-\alpha)(1-\beta)^2$ $+ (1-\alpha)(1-\beta)\alpha\beta$	$MP_{i,t}^{0} =$ $\alpha^2\beta(1-\beta)$ $+ (1-\alpha)^2\beta(1-\beta)$ $+ (\alpha\beta)^2 + [\alpha(1-\beta)]^2$ $+ [(1-\alpha)\beta]^2$ $+ [(1-\alpha)(1-\beta)]^2$ $+ \alpha^2\beta(1-\beta)$ $+ (1-\alpha)^2\beta(1-\beta)$	$MP_{i,t}^{+s} =$ $(1-\alpha)(1-\beta)\alpha\beta$ $+ (1-\alpha)\alpha\beta^2 +$ $\alpha(1-\alpha)(1-\beta)^2$ $+ (1-\alpha)(1-\beta)\alpha\beta$	1.00

Given the process of price changes (i.e., returns) is stationary and its mean is:

$$E[\Delta P_{i,t}] = MP_{i,t}^{-s} \cdot (-s_i) + MP_{i,t}^{0} \cdot 0 + MP_{i,t}^{+s} \cdot (+s_i) = 0 \qquad \textbf{(C1)}$$

where $MP_{i,t}^{-s} = MP_{i,t}^{+s}$, $\forall i$.

The variance of securities i:

$$\begin{aligned} Var(\Delta P_{i,t}) &= (-s_i - 0)^2 \cdot MP_{i,t}^{-s} + (0_i - 0)^2 \cdot MP_{i,t}^{0} + (+s_i - 0)^2 \cdot MP_{i,t}^{+s} \\ &= 2s_i^2 MP_{i,t}^{-s} \end{aligned} \qquad \textbf{(C2)}$$

The covariance of the successive price changes between securities i and j is:

$$\begin{aligned} &Cov(\Delta P_{i,t}, \Delta P_{j,t+1}) \\ &= P_{j=-s}^{i=-s} \cdot (-s_i) \cdot (-s_j) + P_{j=+s}^{i=-s} \cdot (-s_i) \cdot (s_j) + P_{j=-s}^{i=+s} \cdot (s_i) \cdot (-s_j) + P_{j=+s}^{i=+s} \cdot (s_i) \cdot (s_j) \\ &= 0, \end{aligned} \qquad \textbf{(C3)}$$

where

$$P_{j=-s}^{i=-s} = P_{j=+s}^{i=-s} = P_{j=-s}^{i=+s} = P_{j=+s}^{i=+s} = P. \qquad \textbf{Q.E.D.}$$

APPENDIX D: DERIVATION OF COVARIANCE COMPONENT $Cov(R_t^q, R_{t+1}^*)$

In Eq.(10), the portfolio serial return covariance has four covariance components. We believe to illustrate one of them is enough to understand the derivation of others. We choose to derive the covariance component of the lead-lag portfolio liquidity return and the portfolio true return, $Cov(R_t^q, R_{t+1}^*)$. From Eq.(10), we have

$$\begin{aligned} &Cov(R_t, R_{t+1}) \\ &= Cov(R_t^*, R_{t+1}^*) + Cov(R_t^*, R_{t+1}^q) + Cov(R_t^q, R_{t+1}^*) + Cov(R_t^q, R_{t+1}^q), \end{aligned} \qquad \textbf{(10)}$$

where R_t is the portfolio return, R_t^* is the portfolio true return of the intrinsic value, and R_t^q is the portfolio liquidity return at time t,

$$
\begin{aligned}
Cov(R_t^q, R_{t+1}^*) &= \sum_{i=1}^{n} Cov(\delta_{i,t}, r_{i,t+1}^*) + \sum_{i=1}^{n}\sum_{j=1, j\neq i}^{n} Cov(\delta_{i,t}, r_{j,t+1}^*) \\
&= \sum_{i=1}^{n} Cov[\ln(1+s_{i,t}\cdot Q_{i,t}) - \ln(1+s_{i,t-1}\cdot Q_{i,t-1}), \ln(1+s_{i,t}^{AS}\cdot Q_{i,t}+\varepsilon_{i,t+1})] \\
&\quad + \sum_{i=1}^{n}\sum_{j=1, j\neq i}^{n} Cov[\ln(1+s_{i,t}\cdot Q_{i,t}) - \ln(1+s_{i,t-1}\cdot Q_{i,t-1}), \ln(1+s_{j,t}^{AS}\cdot Q_{j,t}+\varepsilon_{j,t+1})] \\
&\approx \sum_{i=1}^{n} Cov(s_{i,t}\cdot Q_{i,t}, s_{i,t}^{AS}\cdot Q_{i,t}+\varepsilon_{i,t+1}) + \sum_{i=1}^{n} - Cov(s_{i,t-1}\cdot Q_{i,t-1}, s_{i,t}^{AS}\cdot Q_{i,t}+\varepsilon_{i,t+1}) \\
&\quad + \sum_{i=1}^{n}\sum_{j=1, j\neq i}^{n} Cov(s_{i,t}\cdot Q_{i,t}, s_{j,t}^{AS}\cdot Q_{j,t}+\varepsilon_{j,t+1}) \\
&\quad + \sum_{i=1}^{n}\sum_{j=1, j\neq i}^{n} - Cov(s_{i,t-1}\cdot Q_{i,t-1}, s_{j,t}^{AS}\cdot Q_{j,t}+\varepsilon_{j,t+1}) \\
&= \sum_{i=1}^{n} [Var(s_{i,t}^{AS}\cdot Q_{i,t}) + Cov(s_{i,t}^{GP}\cdot Q_{i,t}, s_{i,t}^{AS}\cdot Q_{i,t})] \\
&\quad + \sum_{i=1}^{n} - [Cov(s_{i,t-1}^{AS}\cdot Q_{i,t-1}, s_{i,t}^{AS}\cdot Q_{i,t}) + Cov(s_{i,t-1}^{GP}\cdot Q_{i,t-1}, s_{i,t}^{AS}\cdot Q_{i,t})] \\
&\quad + \sum_{i=1}^{n}\sum_{j=1, j\neq i}^{n} [Cov(s_{i,t}^{AS}\cdot Q_{i,t}, s_{j,t}^{AS}\cdot Q_{j,t}) + Cov(s_{i,t}^{GP}\cdot Q_{i,t}, s_{j,t}^{AS}\cdot Q_{j,t})] \\
&\quad + \sum_{i=1}^{n}\sum_{j=1, j\neq i}^{n} - [Cov(s_{i,t-1}^{AS}\cdot Q_{i,t-1}, s_{j,t}^{AS}\cdot Q_{j,t}) + Cov(s_{i,t-1}^{GP}\cdot Q_{i,t-1}, s_{j,t}^{AS}\cdot Q_{j,t})]
\end{aligned}
\tag{13}
$$

where $s_{i,t} = s_{i,t}^{AS} + s_{i,t}^{GP}, \forall i$.

APPENDIX E: POTENTIAL BIASES IN RETURN COVARIANCES BY USING MID-POINT RETURN AS A PROXY FOR TRUE RETURN

If the mid-point price (return) were different from the true value (return) of a security, then what kind of potential bias would it appear in the return covariances? Define $r_{i,t}^M = r_{i,t}^* + \lambda_{i,t}$, and $\delta_{i,t}^M = r_{i,t} - r_{i,t}^M = r_{i,t} - (r_{i,t}^* + \lambda_{i,t}) = \delta_{i,t} - \lambda_{i,t}$, where $r_{i,t}^*$ is the true return, $r_{i,t}^M$ is the mid-point return, $\delta_{i,t}$ is the theoretical liquidity return, $\delta_{i,t}^M$ is the empirical liquidity return, and $\lambda_{i,t} \in R$ is a random measurement error of security i between the mid-point return and the true return at time t. The mid-point return autocovariance to proxy Eq.(6) can be expressed as follows:

$$\begin{aligned}&Cov(r_{i,t}^{M}, r_{i,t+1}^{M})\\&= Cov(r_{i,t}^{*} + \lambda_{i,t}, r_{i,t+1}^{*} + \lambda_{i,t+1})\\&= Cov(r_{i,t}^{*}, r_{i,t+1}^{*}) + Cov(r_{i,t}^{*}, \lambda_{i,t+1}) + Cov(\lambda_{i,t}, r_{i,t+1}^{*}) + Cov(\lambda_{i,t}, \lambda_{i,t+1})\\&\approx Cov(r_{i,t}^{*}, r_{i,t+1}^{*}) + Cov(\lambda_{i,t}, \lambda_{i,t+1})\end{aligned} \tag{E1}$$

The first term in Eq.(E1) is Eq.(6). The magnitudes of $Cov(r_{i,t}^{*}, \lambda_{i,t+1})$ and $Cov(\lambda_{i,t}, r_{i,t+1}^{*})$ are likely trivial because the true return of security i is assumed to be independent to the measurement error $\lambda_{i,t}$ at any time point t. There are three possible bias scenarios in $Cov(r_{i,t}^{M}, r_{i,t+1}^{M})$ induced by the measurement error $\lambda_{i,t}$:

1. $Cov(\lambda_{i,t}, \lambda_{i,t+1}) > 0$:

If the ask (bid) price has a systematic tendency due to, such as, the phenomenon of a buyer- (seller-) initiated trade followed by another buyer- (seller-) initiated trade (e.g., Huang and Stoll, 1997) and if this kind of occurrence were purely driven by liquidity, then the true price would remain constant but the ask (bid) price keeps rising (declining), which lead to a higher (lower) mid-point price and produce a positive $Cov(\lambda_{i,t}, \lambda_{i,t+1})$. This would create an upward bias in estimating $Cov(r_{i,t}^{*}, r_{i,t+1}^{*})$ by using $Cov(r_{i,t}^{M}, r_{i,t+1}^{M})$ as a proxy.

2. $Cov(\lambda_{i,t}, \lambda_{i,t+1}) < 0$:

In Easley and O'Hara (1987, Eqs.(5) and (6)), they show that when the difference between the seller-initiated posterior and prior probabilities of bad (good) news is greater than the difference between the buyer-initiated posterior and prior probabilities of bad (good) news, the bid spread is greater than the ask spread. In this case, the mid-point price $p_{i,t}^{M}$ is lower than the true value $p_{i,t}$, i.e., when $p_{i,t}^{M} < p_{i,t}$, it produces a positive measurement error $\lambda_{i,t}$. In contrast, when the difference between the seller-initiated posterior and prior probabilities of bad (good) news is smaller than the difference between the buyer-initiated posterior and prior probabilities of bad (good) news, the bid spread is smaller than the ask spread. In this case, when $p^M_{i,t} > p_{i,t}$, it produces a negative measurement error $\lambda_{i,t}$. If the changes of the consecutive ask and bid quotes alternate between these two kinds of case, then we would observe a negative $Cov(\lambda_{i,t}, \lambda_{i,t+1})$.

3. $Cov(\lambda_{i,t}, \lambda_{i,t+1}) = 0$:

In this scenario, it implies that the measurement error $\lambda_{i,t}$ itself is uncorrelated in time series, at least at lag one. The rationale is that the mid-point price (return) may be different from the true value (return) of the security, but this measurement error $\lambda_{i,t}$ may contribute little significance at the correlation dimension, if it is idiosyncratic.

The above analysis can also apply to the rest of covariances at the individual security as well as portfolio levels. By assuming the theoretical liquidity return $\delta_{i,t}$ is also independent to the measurement error $\lambda_{i,t}$ at any time point, similar biases may also appear in the other three empirical decomposed return autocovariances that are proxies for Eqs (7), (8), and (9):

$$Cov(r^M_{i,t}, \delta^M_{i,t+1}) \approx Cov(r^*_{i,t}, \delta_{i,t+1}) - Cov(\lambda_{i,t}, \lambda_{i,t+1}) \quad \textbf{(E2)}$$

$$Cov(\delta^M_{i,t}, r^M_{i,t+1}) \approx Cov(\delta_{i,t}, r^*_{i,t+1}) - Cov(\lambda_{i,t}, \lambda_{t+1}) \quad \textbf{(E3)}$$

$$Cov(\delta^M_{i,t}, \delta^M_{i,t+1}) \approx Cov(\delta_{i,t}, \delta_{i,t+1}) + Cov(\lambda_{i,t}, \lambda_{t+1}) \quad \textbf{(E4)}$$

The first terms in Eqs.(E2), (E3), and (E4) correspond to Eqs.(7), (8), and (9). The second terms are the biases induced by the measurement error λ.

At the portfolio level, the portfolio mid-point return autocovariance to proxy Eq.(11) can be expressed as follows:

$$\begin{aligned}
&Cov(R_t^M, R_{t+1}^M) \\
&= \sum_{i=1}^{n} Cov(r_{i,t}^M, r_{i,t+1}^M) + \sum_{i=1}^{n} \sum_{j=1, j\neq i}^{n} Cov(r_{i,t}^M, r_{j,t+1}^M) \\
&= \sum_{i=1}^{n} Cov(r_{i,t}^* + \lambda_{i,t}, r_{i,t+1}^* + \lambda_{i,t+1}) + \sum_{i=1}^{n} \sum_{j=1, j\neq i}^{n} Cov(r_{i,t}^* + \lambda_{i,t}, r_{j,t+1}^* + \lambda_{j,t+1}) \\
&\approx \sum_{i=1}^{n} Cov(r_{i,t}^*, r_{i,t+1}^*) + \sum_{i=1}^{n} \sum_{j=1, j\neq i}^{n} Cov(r_{i,t}^*, r_{j,t+1}^*) \\
&\quad + \sum_{i=1}^{n} Cov(\lambda_{i,t}, \lambda_{i,t+1}) + \sum_{i=1}^{n} \sum_{j=1, j\neq i}^{n} Cov(\lambda_{i,t}, \lambda_{j,t+1})
\end{aligned} \qquad \textbf{(E5)}$$

The first two terms in Eq.(E5) correspond to Eq.(11) and the remaining two terms are the biases induced by the measurement error λ.

As in Eq.(E1), $\sum_{i=1}^{n} Cov(\lambda_{i,t}, \lambda_{i,t+1})$ could produce a positive, negative, or no bias in $Cov(R_t^M, R_{t+1}^M)$. $\sum_{i=1}^{n} \sum_{j=1, j\neq i}^{n} Cov(\lambda_{i,t}, \lambda_{j,t+1})$ also has three bias scenarios:

1. $\sum_{i=1}^{n} \sum_{j=1, j\neq i}^{n} Cov(\lambda_{i,t}, \lambda_{j,t+1}) > 0$ **:**

If the cross-security measurement errors are lead-lag correlated due to, such as, the herding phenomenon of a buyer- (seller-) initiated trade of security i followed by another buyer- (seller-) initiated trade of security j and if this kind of occurrence were purely driven by liquidity for securities involved, then the true prices would remain constant but ask (bid) prices of involved securities tend to go up (down), which could make $\sum_{i=1}^{n} \sum_{j=1, j\neq i}^{n} Cov(\lambda_{i,t}, \lambda_{j,t+1})$ positive.

2. $\sum_{i=1}^{n} \sum_{j=1, j\neq i}^{n} Cov(\lambda_{i,t}, \lambda_{j,t+1}) < 0$ **:**

Similar to (i), only it is a reversal-herding phenomenon. If a buyer- (seller-) initiated trade of security i tends to be followed by another seller- (buyer-) initiated trade of security j and if this kind of occurrence were purely driven by liquidity for securities involved, then the true prices would remain constant but ask (bid) prices of some securities tend to go up (down) and others tend to go down (up), which could make $\sum_{i=1}^{n}\sum_{j=1, j\neq i}^{n} Cov(\lambda_{i,t}, \lambda_{j,t+1})$ negative.

3. $\sum_{i=1}^{n}\sum_{j=1, j\neq i}^{n} Cov(\lambda_{i,t}, \lambda_{j,t+1}) = 0$ **:**

The measurement errors $\lambda_{i,t}, \forall i$, have no lead-lag cross-correlation, at least at lag one. Again, the mid-point price (return) may be different from the true value (return) of security i, $\forall i$, but their measurement errors $\lambda_{i,t}, \forall i$, may contribute little significance at the cross-security lead-lag correlation dimension.

By assuming the liquidity return δ_i of security i is also independent to the measurement error λ_j of any security j, including security i itself, at any time point, similar biases may also appear in Eqs.(E6), (E7), and (E8) that are proxies for Eqs. (12), (13), and (14):

$$
\begin{aligned}
&Cov(R_t^M, R_{t+1}^{q^M}) \\
&\approx \sum_{i=1}^{n} Cov(r_{i,t}^*, \delta_{i,t+1}) + \sum_{i=1}^{n}\sum_{j=1, j\neq i}^{n} Cov(r_{i,t}^*, \delta_{j,t+1}) \\
&\quad - \sum_{i=1}^{n} Cov(\lambda_{i,t}, \lambda_{i,t+1}) - \sum_{i=1}^{n}\sum_{j=1, j\neq i}^{n} Cov(\lambda_{i,t}, \lambda_{j,t+1})
\end{aligned}
\tag{E6}
$$

$$
\begin{aligned}
&Cov(R_t^{q^M}, R_{t+1}^M) \\
&\approx \sum_{i=1}^{n} Cov(\delta_{i,t}, r_{i,t+1}^*) + \sum_{i=1}^{n}\sum_{j=1, j\neq i}^{n} Cov(\delta_{i,t}, r_{j,t+1}^*) - \sum_{i=1}^{n} Cov(\lambda_{i,t}, \lambda_{i,t+1}) \\
&\quad - \sum_{i=1}^{n}\sum_{j=1, j\neq i}^{n} Cov(\lambda_{i,t}, \lambda_{j,t+1})
\end{aligned}
\tag{E7}
$$

$$Cov(R_t^{q^M}, R_{t+1}^{q^M})$$
$$\approx \sum_{i=1}^{n} Cov(\delta_{i,t}, \delta_{i,t+1}) + \sum_{i=1}^{n} \sum_{j=1, j \neq i}^{n} Cov(\delta_{i,t}, \delta_{j,t+1}) \quad \textbf{(E8)}$$
$$+ \sum_{i=1}^{n} Cov(\lambda_{i,t}, \lambda_{i,t+1}) + \sum_{i=1}^{n} \sum_{j=1, j \neq i}^{n} Cov(\lambda_{i,t}, \lambda_{j,t+1})$$

The first two terms in Eqs.(E6), (E7), and (E8) correspond to Eqs.(12), (13), and (14). The remaining two terms are the biases induced by the measurement error λ.

連續競價市場中流動性之資訊角色

王子真*

摘要

於連續競價市場中，探索流動性變數在私屬資訊傳遞上所扮演之角色。我們發覺個別證券價值對於私屬資訊的更新是呈現於前期價差中之逆選擇成本元素與後期真實報酬率之間的互動。但是如果因此以為該證券價值之更新就只是由於其自己前期價差中之逆選擇成本元素所造成，這觀點可能過於狹隘。於投資組合的層面可知，個別證券價值之更新是由於投資組合中所有成份股的前期與同期價差中之逆選擇成本元素交互造成。此現象稱之為「私屬資訊的共同性」。

關鍵詞彙：流動性，資訊不對稱，買賣價差

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